High Dynamic Range Spectral Estimation for Incomplete Time Series

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Abstract

Estimating complex spectra is a widespread operation in signal processing and in some applications a high dynamic range, which requires low sidelobe levels, is essential. For data with uniform sample spacing, weightings are commonly applied to Fourier transforms to suppress sidelobes, increasing dynamic range at the cost of some loss of spectral resolution. However, if a significant proportion of the samples lack data, conventional weightings suffer from high sidelobe levels. While a wide range of linear and non-linear techniques has been proposed to tackle this problem, they are not suitable in applications, such as Synthetic Aperture Radar (SAR), which require a high dynamic range. We propose criteria that allow optimal weights to be computed for any pattern of sample times. The resulting weighted Fourier transform has advantages of inexpensive computation, easily understood characteristics arising from its linearity, a position-independent impulse response and importantly the transform is phase preserving. Potential applications include wideband radar, where spectral gaps are needed for coexistence with other systems; multifunction radar, where imaging is interrupted by other tasks; and bistatic radar, where the spectrum of a transmitter of opportunity may be incomplete. In the context of SAR, high dynamic range, phase preserving spectral estimation supports post-processing such as interferometry and coherent change detection.

Objectives

- Achieve high-dynamic range by suppressing sidelobes
- Optimise resolution, while avoiding peak splitting
- Preserve phase of spectral estimate

Approach

- Estimate the spectrum with weighted FFT & linear phase correction
- Minimise the sidelobe power
- Constrain the noise gain
- Constrain the mainlobe gain, curvature and phase variation

For signal $s(t)$ sampled at times t_j, spectral estimate is

$$S(F) = \sum_j w_j \, s(t_j) \, e^{-2\pi i F (t_j - T)}$$

We need to choose weights w_j and centre time T

Analysis

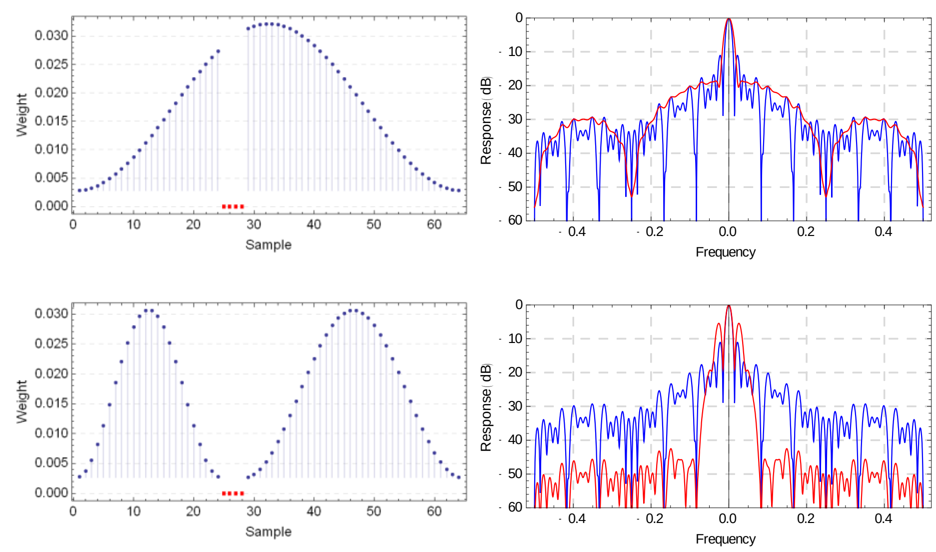
- Sidelobe power and noise gain are quadratic forms in the weights
- Linear constraint on weights can:
  - control mainlobe gain and curvature
  - Zero phase variation can be zeroed up to 3rd derivative

Minimise	$\sum_{i,j} A_{ij} w_i w_j$ wrt w_i, T	(sidelobe power)
subject to	$\sum_j w_j^2 \le G^2$	(noise gain limit)
	$\sum_j w_j = 1$	(constant mainlobe gain)
	$\sum_j w_j t_j = T$	(zero linear phase gradient)
	$\sum_j w_j t_j^2 - T^2 = K$	(peak curvature)
	$\sum_j w_j t_j^3 - 3KT - T^3 = 0$	(zero cubic phase gradient)

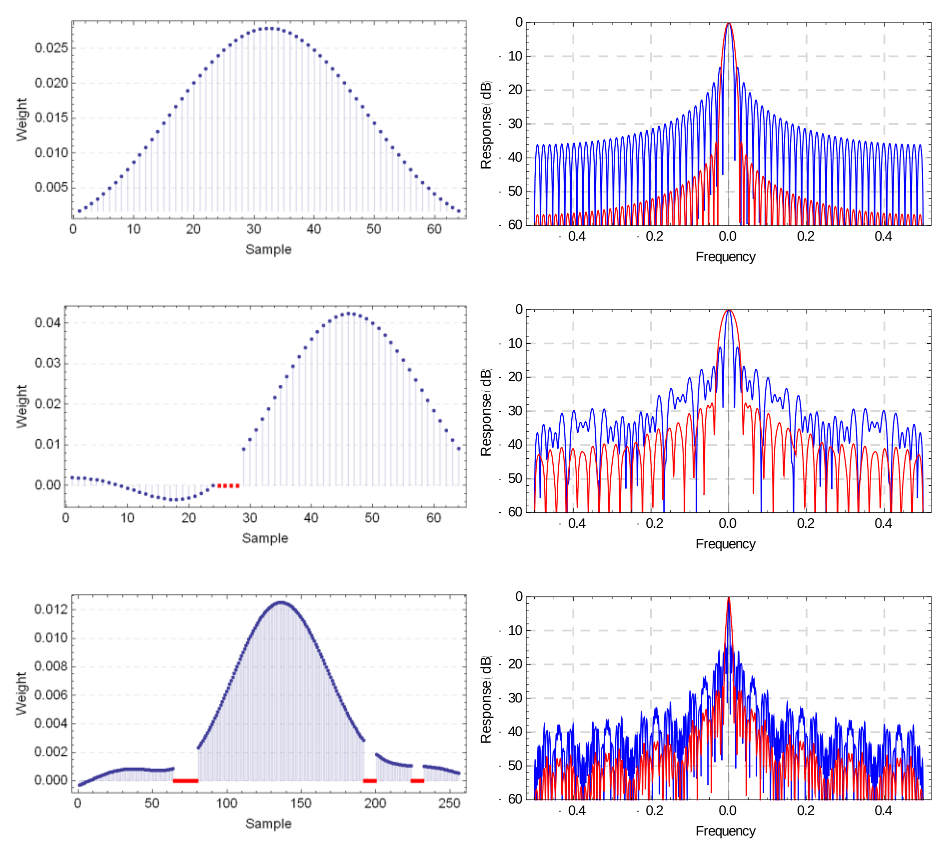
Example

Ad hoc weightings give high sidelobe levels or split peaks

Weighted spectra **Unweighted spectra**

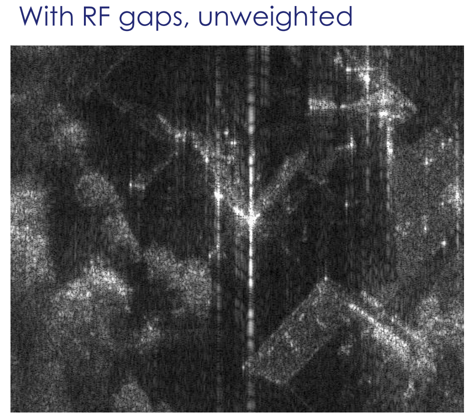


High-Dynamic Range Results

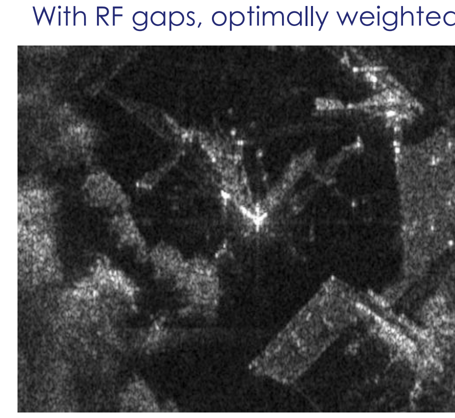


SAR Image Results

With RF gaps, unweighted



With RF gaps, optimally weighted



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