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Transmit Adaptivity in Radar

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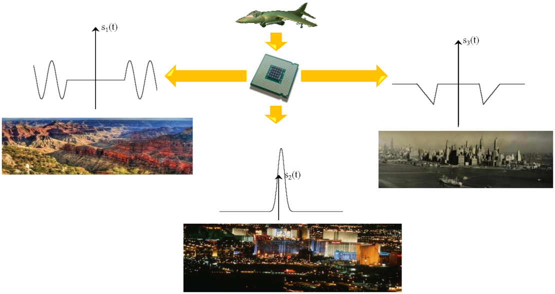
Outline

1. Transmit Adaptivity in Radar
2. Radar Waveform Design for Spectral Coexistence
3. Some Results
4. Conclusions and Future Researches
5. References

Transmit Adaptivity: Introduction & Motivation

Radar performance is highly **dependent** on the **probing waveform**.



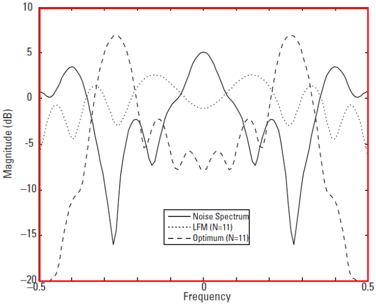
Waveform design can be formulated as a **Constrained Optimization Problem**.

Objective **Maximize Radar Performance**

Subject to **Interference $\leq$ Acceptable Level**
Similarity Constraint
Energy Constraint
Knowledge-aided Constraints

Optimizing Fast-Time Modulation

Benefits of **tailoring the transmit waveform** (fast-time modulation) to account for a colored noise RF interference source.



Additional context-dependent constraints can be also forced to the radar waveform.

This shaping technique can be also exploited to **control the impact of radar on other communication systems**.

- A. De Maio, S. De Nicola, Z.-Q. Luo and S. Zhang, "*Design of Phase Codes for Radar Performance Optimization with a Similarity Constraint*", IEEE Transactions on Signal Processing, February 2009.
- J. R. Guerci, "*Cognitive Radar, the Knowledge-Aided Fully Adaptive Approach*", 2010.

Spectral Coexistence

Spectrally Crowded Environments

Coexistence among radar and telecommunication systems is currently becoming one of the **challenging research topics** in both radar and communication communities.

"The desire to autonomously anticipate, find, fix, track, target, engage and assess anything, anytime, anywhere in spectrally-dense environments will require changes to how build, modify, and deploy radar and radio frequency systems." M. Wicks 2010.

It is thus **mandatory** the development of **advanced radar signals** ensuring **compatibility** with the surrounding electromagnetic radiators, namely keeping acceptable the mutual interference induced on frequency overlaid systems.

Signal Model

Let us consider a monostatic **radar system** transmitting a signal composed of N **sub-pulses** and denote by

$$\mathbf{c} = [c(1), \ldots, c(N)]^T \in \mathbb{C}^N$$

the N-dimensional fast-time radar code. Thus, the N-dimensional column vector $\mathbf{v} \in \mathbb{C}^N$ of the observations, from the range-azimuth cell under test, can be expressed as:

$$\mathbf{v} = \alpha \mathbf{c} + \mathbf{n}.$$

- α is a complex parameter accounting for channel propagation and backscattering effects from the target within the range-azimuth bin of interest;
- $\mathbf{n}$ is the N-dimensional column vector containing the **filtered disturbance echo samples**:
    1. it accounts for both **white internal thermal noise** as well as **interfering signals sharing the same frequencies as the radar of interest**;
    2. it is modeled as a **complex, zero-mean, circular Gaussian random vector** sharing the covariance matrix $\mathbf{M}$.

Cooperative Radiators & Induced Interference

Cooperative radiator working over a **frequency band** $\Omega_k = [f_1^k, f_2^k]$.

Radar energy radiated on Ω_k

$$\mathbf{c}^\dagger \mathbf{R}_I^k \mathbf{c}$$

where $\mathbf{R}_I^k$ depends on Ω_k.

To guarantee **spectral compatibility** with K overlayed radiators, the radar has to **control the energy produced on the shared frequency bands**.

- **Local control**:

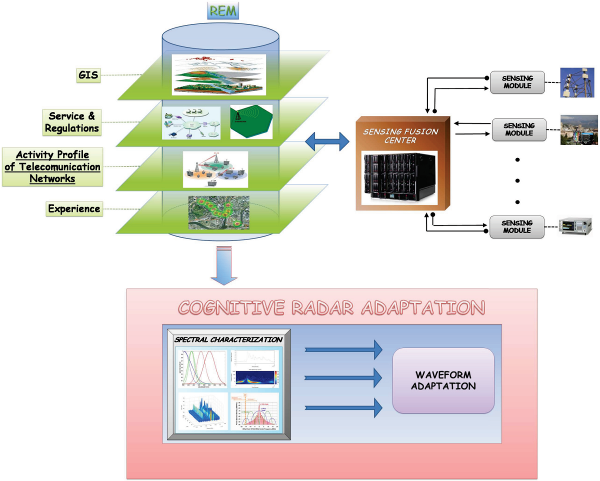
  $$\mathbf{c}^\dagger \mathbf{R}_I^k \mathbf{c} \leq E_I^k, \ k = 1, \ldots, K$$

  - E_I^k is the **amount of allowed interference level** on the k-th band, $k = 1, \ldots, K$.
- **Global control**:

  $$\mathbf{c}^\dagger \mathbf{R}_I \mathbf{c} \leq E_I,$$

  - $\mathbf{R}_I = \sum_{k=0}^{K} w_k \mathbf{R}_I^k$, with $w_k \geq 0$, $k = 0, \ldots, K$, reflects the importance of a given radiator;
  - E_I is the **global allowed interference level**.

Cognitive Spectrum Awareness



REM

GIS

Service & Regulations

Activity Profile of Telecomunication Networks

Experience

SENSING FUSION CENTER

SENSING MODULE

SENSING MODULE

SENSING MODULE

COGNITIVE RADAR ADAPTATION

SPECTRAL CHARACTERIZATION

WAVEFORM ADAPTATION

Radio Environment Map (REM) represents the key to gain spectrum cognizance which is at the base of an **intelligent and agile spectrum management**.

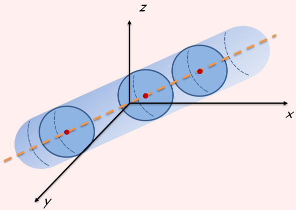
Waveform Design: Objective Function & Constraints 1/2

- **Optimizing the detection performance**, through the maximization of the **Signal to Interference plus Noise Ratio (SINR)**, namely

$$\mathrm{SINR} = |\alpha|^2 \mathbf{c}^\dagger \mathbf{R}\mathbf{c},$$

where $\mathbf{R} = \mathbf{M}^{-1}$.

- Forcing **desirable radar features** to the transmitted waveform accounting for an **energy constraint** and a **generalized similarity constraint** with a prescribed waveform $\mathbf{c}_0$.



$$\|\mathbf{c} - \alpha_{c_0}\mathbf{c}_0\|^2 \leq \epsilon$$
$$|\alpha_{c_0}|^2 \leq 1$$

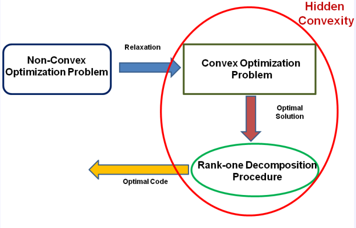
- Providing a control on the **interference energy** produced on shared bands.

Waveform Design: Objective Function & Constraints 2/2

The **waveform design problems** can be formulated as:

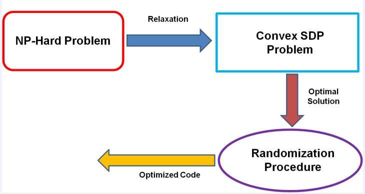
Global Control

$$\begin{cases} \max\limits_{\mathbf{c}\in\mathbb{C}^N,\alpha_{c_0}\in\mathbb{C}} & \mathbf{c}^\dagger\mathbf{R}\mathbf{c} \\ \text{s.t.} & \|\mathbf{c}\|^2\le 1 \\ & \mathbf{c}^\dagger\mathbf{R}_I\mathbf{c}\le E_I \\ & \|\mathbf{c}-\alpha_{c_0}\mathbf{c}_0\|^2\le\epsilon \\ & |\alpha_{c_0}|^2\le 1 \end{cases}.$$


Hidden Convexity
Non-Convex Optimization Problem
Relaxation
Convex Optimization Problem
Optimal Solution
Rank-one Decomposition Procedure
Optimal Code

Local Control

$$\begin{cases} \max\limits_{\mathbf{c}\in\mathbb{C}^N,\alpha_{c_0}\in\mathbb{C}} & \mathbf{c}^\dagger\mathbf{R}\mathbf{c} \\ \text{s.t.} & \|\mathbf{c}\|^2\le 1 \\ & \mathbf{c}^\dagger\mathbf{R}_I^k\mathbf{c}\le E_I^k, \quad i=1,\ldots K \\ & \|\mathbf{c}-\alpha_{c_0}\mathbf{c}_0\|^2\le\epsilon \\ & |\alpha_{c_0}|^2\le 1 \end{cases}.$$


NP-Hard Problem
Relaxation
Convex SDP Problem
Optimal Solution
Randomization Procedure
Optimized Code

Some Results

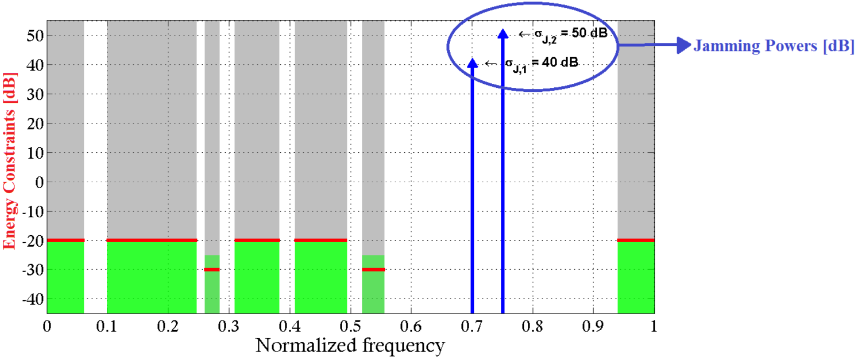
The baseband equivalent transmitted signal has a **two-sided bandwidth** of 810 kHz and a Nyquist sampling frequency is used. The **disturbance covariance matrix** is modeled as

$$\mathbf{M} = \underbrace{\mathbf{I}}_{\textbf{white noise}} + \underbrace{\sum_{k=1}^{7} \frac{\sigma_{I,k}}{f_2^k - f_1^k} \mathbf{R}_I^k}_{\substack{\textbf{licensed coexisting} \\ \textbf{telecommunication networks}}} + \underbrace{\sum_{k=1}^{2} \sigma_{J,k} \mathbf{R}_{J,k}}_{\substack{\textbf{unlicensed narrowband} \\ \textbf{continuous jammers}}}$$

- $\sigma_{J,k}$ and $\sigma_{I,k}$ account for the energy of the k-th active jammer and the energy of the k-th coexisting telecommunication network operating on the **normalized frequency band** Ω_k ($\sigma_{I,k} = 10$ dB, $k = 1, \dots, 7$, $\sigma_{J,1} = 40$ dB, $\sigma_{J,2} = 50$ dB);
- $\mathbf{R}_{J,k}$ is the **normalized covariance matrix** of the k-th **active unlicensed narrowband jammer**.

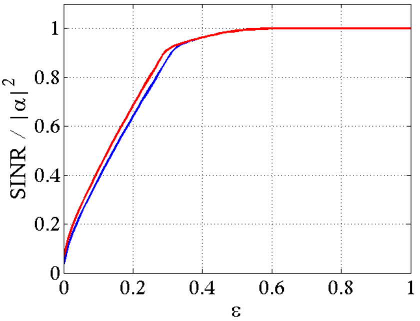
Some Results 2/5

Spectral mask & **Jammer Doppler** locations

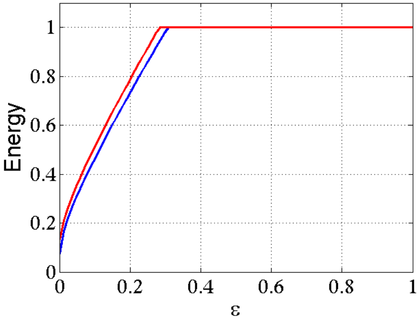


As to the **reference code** $\mathbf{c}_0$, a **unitary norm LFM pulse** with a duration of 200 μs and a chirp rate $K_s = (750 \times 10^3)/(200 \times 10^{-6})$ Hz/s is employed.

Some Results



Normalized SINR versus ϵ.



Code energy versus ϵ.

Legend: **Global Design**;
Local Design.

Some Results

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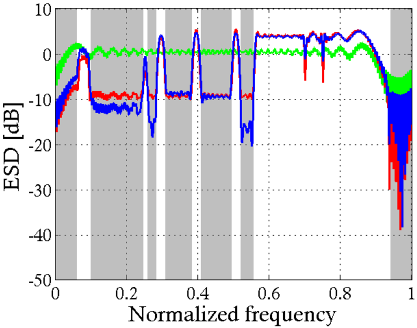
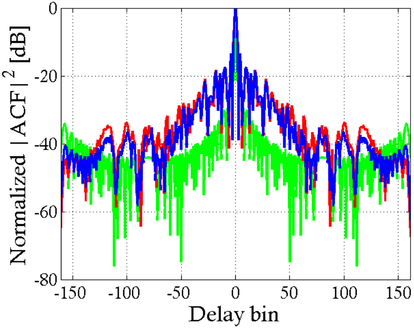


ESD versus normalized frequency considering $\epsilon = 0.31$.



Squared modulus of ACF versus delay bin considering $\epsilon = 0.31$.

Legend: **Reference Code**;
Global Design;
Local Design.

Some Results

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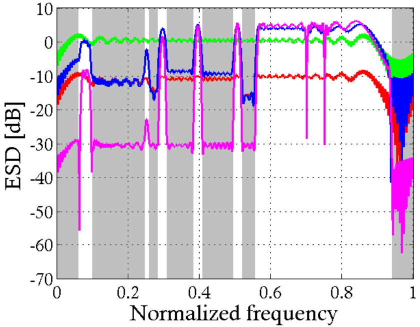
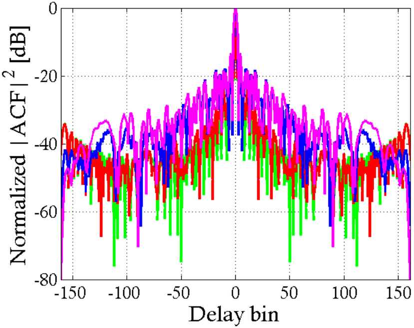


ESD versus normalized frequency.



Squared modulus of ACF versus delay bin.

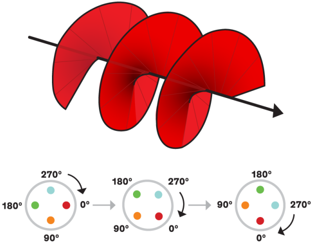
Legend: **Reference Code**;
Local Design for $\epsilon = 0.001$;
Local Design for $\epsilon = 0.322$;
Local Design for $\epsilon = 0.584$.

Conclusions and Future Researches

- **Transmit Adaptivity** in Radar has been discussed.
- **Synthesis of radar waveforms** in **spectrally crowded environment** has been presented and analyzed.

Possible **future research tracks**: development of robust frameworks to contrast **transmitter impurities** and the fully exploitation of the available **multiple dimensions**:

- **polarization**;
- **space**;
- **frequency**;
- **orbital angular momentum**;
- ...



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- A. Aubry, V. Carotenuto and A. De Maio, “Forcing Multiple Spectral Compatibility Constraints in Radar Waveforms”, IEEE Signal Processing Letters, Vol. 23, No. 4, pp 483-487, April 2016.
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