Scanning Emitter TMA by Two Fixed Observers using Time of Interception

Zhang Yifei[1], Zhang Min[2], Guo Fucheng[2]

1. Rocket Force University of Engineering, Xi'an, P. R. China
2. National University of Defense Technology, Changsha, P. R. China

Introduction

Determining the position and velocity of a moving source by one or more observers, which is also referred as target motion analysis (TMA), is essential both in military and civilian applications. However, the traditional TMA methods cannot be directly used for the scanning emitter, such as the mechanically scanning radar, from which only the measurements named as time of interception (TOI) can be obtained.

The TOI-based tracking of a moving scanning emitter with known fixed scan rate by two fixed observers is investigated in this paper, and our main works are as follows,

1. The multiple scan cycles TMA model is established by transforming the TOI to the direction difference of arrival (**DDOA**).
2. A pseudo-linear least square (PLS) estimator is firstly proposed to obtain the initial position and velocity. And an instrumental variable (**IV**) estimator is finally proposed to reduce the estimation bias in the PLS estimator.
3. Simulation results show that the IV estimator attains performance close to the Cramer-Rao lower bound (**CRLB**) at moderate noise levels.

Methods

1. Problem formulation

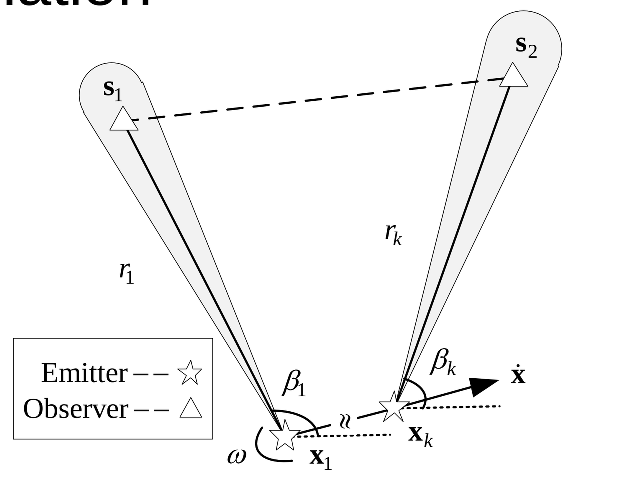

Fig .1. Tracking a moving scanning emitter with two fixed observers.

As shown in *Fig. 1*, the antenna of an moving emitter $\mathbf{x}$ is scanning with a known constant scan rate ω. Its main beam periodically sweeps across two fixed observers $\mathbf{s}_1$, $\mathbf{s}_2$, which intercept the signals and record the interception time of the beam peaks. Assuming that at time t_k, one of the observers intercepts the main beam signal. The TOI measurements of the scanning signal is

$$\hat{t}_k = t_k + \eta_k = (\beta_k - \beta_0)/\omega + \lfloor (k-1)/2 \rfloor T_p + r_k/c + t_0 + \eta_k$$

where c is the speed of light. T_p is the known scanning period. t_0 is the initial transmission time of the scanning signal. r_k denotes the signal propagation range. β_k is the scanning angle. We assume that $\mathbf{s}_1$ intercepts the scanning signal for the first time. Afterward, the two observers intercept the scanning signal alternately without signal loss.

After some approximations, the difference directions of arrival (DDOA), derived from the TOIs, is given by

$$\hat{\beta}_{k1} = \beta_{k1}(\mathbf{u}_1) + \varepsilon_{k1} = \arccos\left[(\mathbf{s}_1 - \mathbf{x}_1)^T(\mathbf{s}_n - \mathbf{x}_k)/(r_1 r_k)\right] + \varepsilon_{k1}$$
$$= \omega\hat{t}_{k1} - 2(k-1)\pi = \omega t_{k1} - 2(k-1)\pi + \varepsilon_{k1}$$

where $\mathbf{u}_1 = [\mathbf{x}_1^T \ \dot{\mathbf{x}}^T]^T$ is the unknown to be estimated. ε_{k1} is the DDOA measurement noise. And the transition function is approximated as

$$\mathbf{x}_k = \mathbf{x}_1 + t_{k1}\dot{\mathbf{x}} = \mathbf{x}_1 + \hat{t}_{k1}\dot{\mathbf{x}} - \eta_{k1}\dot{\mathbf{x}} \approx \mathbf{x}_1 + \hat{t}_{k1}\dot{\mathbf{x}}$$

Accordingly, the scanning emitter TMA problem can be transformed to an optimization problem, which is to estimate the initial state $\mathbf{u}_1$ from a batch of DDOA measurements.

2. Estimation Algorithms

By introducing an extra parameter β_1, the nonlinear problem has a pseudo-linear form as

$$\hat{z}_k = \hat{\mathbf{h}}_k \mathbf{\Phi}_{k1}\boldsymbol{\theta}_1 + \nu_k \approx \hat{\mathbf{h}}_k \hat{\mathbf{\Phi}}_{k1}\boldsymbol{\theta}_1 + \nu_k$$

where $\hat{\mathbf{h}}_k = [\sin\hat{\beta}_{k1} \ \ -\cos\hat{\beta}_{k1} \ \ 0 \ \ 0 \ \ y_{On}\cos\hat{\beta}_{k1} - x_{On}\sin\hat{\beta}_{k1}]$,
$\hat{z}_k = x_{On}\cos\hat{\beta}_{k1} + y_{On}\sin\hat{\beta}_{k1}$, $\boldsymbol{\theta}_1 = \mathbf{R}_1[\mathbf{u}_1^T \ \ 1]^T$, $\mathbf{R}_{\beta_1} = \begin{bmatrix} \cot\beta_1 & 1 \\ -1 & \cot\beta_1 \end{bmatrix}$,

$$\mathbf{R}_1 = \begin{bmatrix} \mathbf{R}_{\beta_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{\beta_1} & \\ \hline \mathbf{0} & & \cot\beta_1 \end{bmatrix}, \quad \hat{\mathbf{\Phi}}_{k1} = \begin{bmatrix} \mathbf{I}_2 & \hat{t}_{k1}\mathbf{I}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_2 & \\ \hline \mathbf{0} & & 1 \end{bmatrix}.$$

the PLS estimator of $\boldsymbol{\theta}_1$ is given by

$$\hat{\boldsymbol{\theta}}_{\mathrm{PLS}} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{z}$$

where $\mathbf{z} = [\hat{z}_2 \ \ \hat{z}_3 \ \cdots \ \hat{z}_N]^T$, $\mathbf{H} = [\hat{\mathbf{h}}_2^T \ \ \hat{\mathbf{h}}_3^T \ \cdots \ \hat{\mathbf{h}}_N^T]^T$, $\mathbf{v} = [\nu_2 \ \ \nu_3 \ \cdots \ \nu_N]^T$. Then, the initial state of the emitter can be determined as

$$\begin{cases} \hat{\beta}_1 = \mathrm{arccot}\left(\hat{\boldsymbol{\theta}}_{(3)}\right) \\ \hat{\mathbf{x}} = \hat{\mathbf{R}}_{\beta_1}^T \hat{\boldsymbol{\theta}}_{(1:2)} \sin\hat{\beta}_1 \end{cases}$$

where $\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}_{\mathrm{PLS}}$, $\hat{\mathbf{R}}_{\beta_1} = \mathbf{R}_{\beta_1}|_{\beta_1 = \hat{\beta}_1}$.

The PLS estimator has enormous bias due to the correlations between the measurement matrix and the noises. In order to overcome this difficulty, an instrumental variable (IV) estimator is proposed and is given by

$$\hat{\boldsymbol{\theta}}_{\mathrm{IV}} = (\mathbf{G}^T\mathbf{H})^{-1}\mathbf{G}^T\mathbf{z}$$

where $\mathbf{G}$ is the IV matrix, which is approximated with the following steps,

- the DDOA evaluated from the PLS estimator is given as
$$\hat{\beta}_{\mathrm{PLS},k1} = \beta_k(\hat{\mathbf{u}}_1) - \hat{\beta}_{\mathrm{PLS},1}$$
- the TDOI is estimated by
$$\hat{t}_{\mathrm{PLS},k1} = \hat{\beta}_{\mathrm{PLS},k1}/\omega + \lfloor (k-1)/2 \rfloor T_p$$
- replacing $\hat{\beta}_{k1}$ and $\hat{t}_{k1}$ with $\hat{\beta}_{\mathrm{PLS},k1}$ and $\hat{t}_{\mathrm{PLS},k1}$ in $\mathbf{H}$ gives the IV matrix as
$$\mathbf{G} = \mathbf{H}|_{t_{k1} = \hat{t}_{\mathrm{PLS},k1}, \beta_{k1} = \hat{\beta}_{\mathrm{PLS},k1}}$$

Results

The moving scanning emitter tracking geometry considered in the simulation is shown in *Fig.2*. The simulation results are shown from *Fig. 3* to *Fig. 4*.

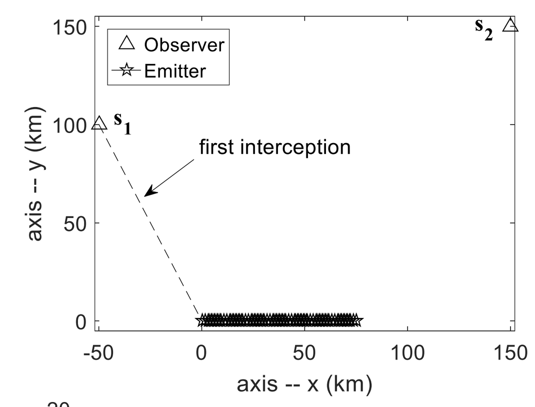

Fig. 2. The geometry of the moving scanning emitter tracking by two fixed observers. The velocity of the emitter is [150 0][T], *and the scan cycle is* 10s.

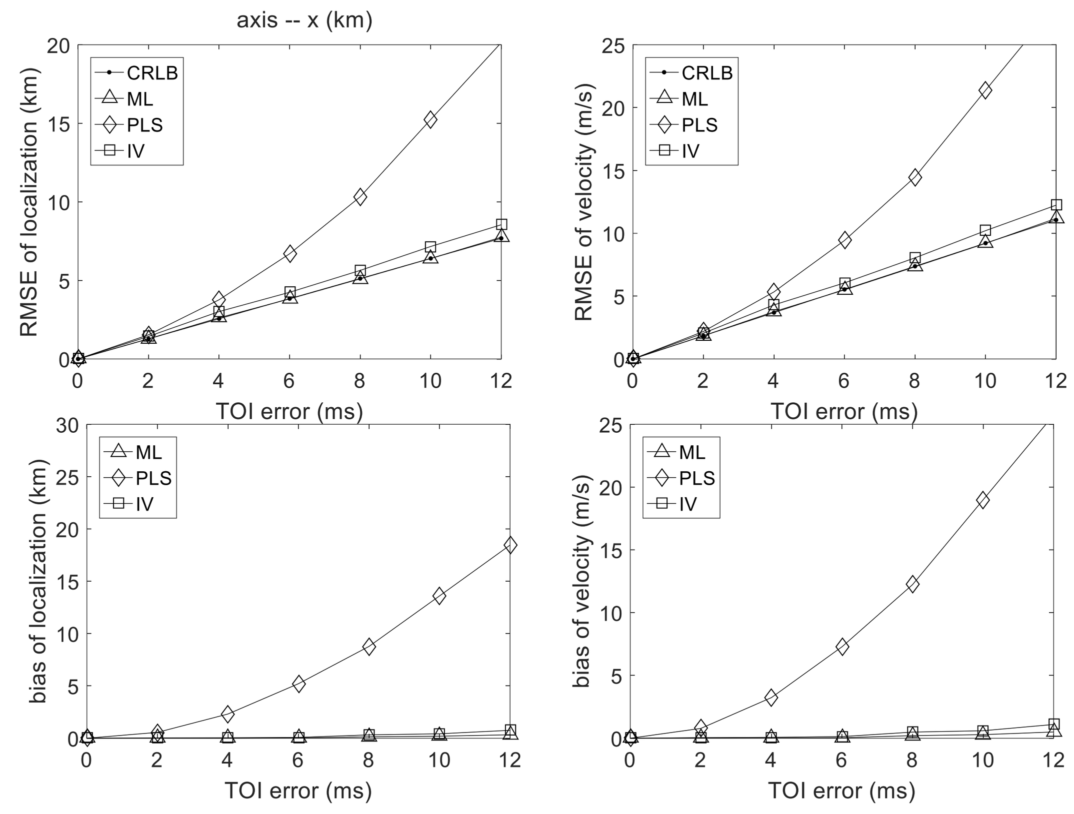

Fig. 3. The RMSE and bias of location and velocity estimation with different noise levels.

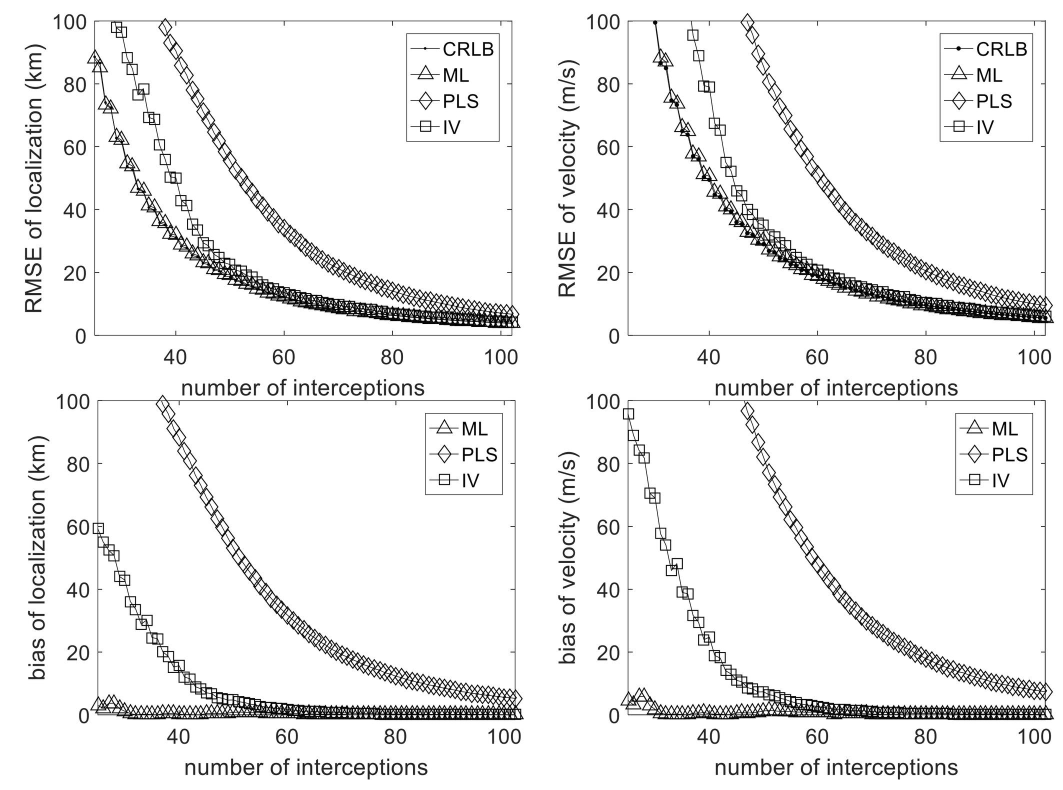

Fig. 4. The RMSE and bias of location and velocity estimation with different numbers of interceptions when $\sigma = 6\text{ms}$

Conclusion

- The moving scanning emitter TMA by two fixed observers has been discussed in this paper. The proposed IV estimator reduces the bias and variance caused by the PLS estimator explicitly. The performance of the IV estimator is comparative to the ML estimator and reaches the CRLB at moderate TOI measurement errors.
- The simulation scenario presented in the paper is suitable for the observers deployed along the seashore and to track the airborne scanning emitter.

Contact: *Zhang Yifei, Information Engineering department, Rocket Force University of Engineering, Xi'an, Shaanxi, 710025, P. R. China*
Email: *littleblack7@126.com*

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