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Joint Sensor Scheduling and Target Tracking with Efficient Bayesian Optimisation

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Outline

I. Motivation

II. Background of Bayesian Optimisation

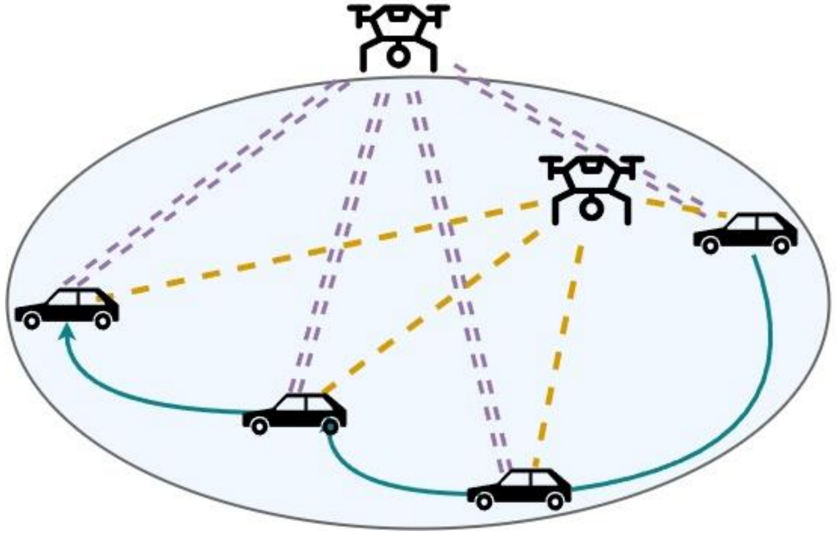
III. Efficient Bayesian Optimisation with Gaussian Process Factorisation

IV. Numerical Results

V. Conclusions and Future Plan

I. Motivation

- Track **without** initial state belief
- Model-**free** approach

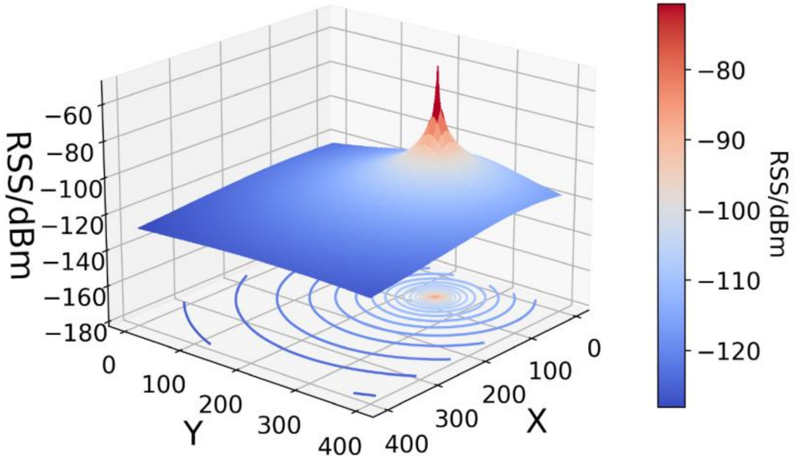


$$y = f(\mathbf{x}_t, t) + \epsilon$$

y: Received signal strength (RSS) measurement

$\mathbf{x}_t$: Location of this measurement at time t

ϵ : Measurement noise, $f(\mathbf{x}_t, t)$:Unknown latent function



Aim	Search and track the target without state transition model and initial state belief, only using the RSS measurement

II. Background of Bayesian Optimisation

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1. **Gaussian Process:** Surrogate model of the unknown function

$$f(\mathbf{x}_t, t) \sim \mathcal{GP}\left(m(\mathbf{x}_t, t), k((\mathbf{x}_t, t), (\mathbf{x}'_t, t'))\right)$$

2. **Acquisition function**: Expected improvement (EI)

Define the object of interest first n_t measurements as $\tau_{n_t} = \max_{i \in n_t} \mathbf{y}_{t_i}$

The EI function can be defined as

$$\begin{aligned} \mathrm{EI}(\mathbf{x}_t, t) &:= \mathbb{E}[[f(\mathbf{x}_t, t) - \tau_{n_t}]^+], \\ &= \underbrace{\sigma(\mathbf{x}_t, t)\phi\left(\frac{\mu(\mathbf{x}_t, t) - \tau_{n_t}}{\sigma(\mathbf{x}_t, t)}\right)}_{\text{Exploration}} + \underbrace{(\mu(\mathbf{x}_t, t) - \tau_{n_t})\Phi\left(\frac{\mu(\mathbf{x}_t, t) - \tau_{n_t}}{\sigma(\mathbf{x}_t, t)}\right)}_{\text{Exploitation}}, \end{aligned}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the probability density function and cumulative density function of the standard Gaussian distribution, respectively.

II. Background of Bayesian Optimisation

The University Of Sheffield.

1. **Gaussian Process:** Surrogate model of the unknown function

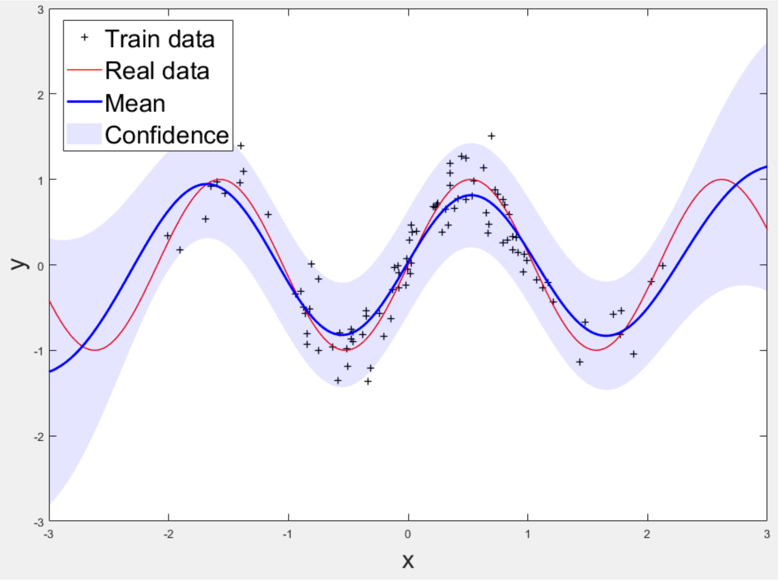
$$f(\mathbf{x}_t, t) \sim \mathcal{GP}\left(m(\mathbf{x}_t, t), k((\mathbf{x}_t, t), (\mathbf{x}'_t, t'))\right)$$

- A Gaussian Process (GP) is a stochastic process defining a distribution over possible functions that fit a set of points.

$$f(x) \sim GP(m(x), k(x, x'))$$

$$m(x_*) = \mathbf{K}_* (\mathbf{K} + \sigma^2 \mathbf{I})^{-1} \mathbf{y}$$

$$k(x, x_*) = \mathbf{K}_{**} - \mathbf{K}_* (\mathbf{K} + \sigma^2 \mathbf{I})^{-1} \mathbf{K}_*^\top$$



- The computational complexity is $O(n^3)$.

II. Background of Bayesian Optimisation

Acquisition function: Expected improvement (EI)

$$\mathrm{EI}(\mathbf{x}_t, t) := \mathbb{E}[[f(\mathbf{x}_t, t) - \tau_{n_t}]^+],$$
$$= \underbrace{\sigma(\mathbf{x}_t, t)\phi\left(\frac{\mu(\mathbf{x}_t, t) - \tau_{n_t}}{\sigma(\mathbf{x}_t, t)}\right)}_{\text{Exploration}} + \underbrace{(\mu(\mathbf{x}_t, t) - \tau_{n_t})\Phi\left(\frac{\mu(\mathbf{x}_t, t) - \tau_{n_t}}{\sigma(\mathbf{x}_t, t)}\right)}_{\text{Exploitation}},$$

- Optimisation problem (sensor management)

  Find the **maximum** of the unknown function

- Exploration-exploitation (EE) tradeoff
  - Where and when to place the UAV to measure RSS
  - Locate the target with **minimum** number of measurements

III. Efficient BO with GP Factorisation

Gaussian Process: Surrogate model of the unknown function

$$f(\mathbf{x}_t, t) \sim \mathcal{GP}\left(m(\mathbf{x}_t, t), k((\mathbf{x}_t, t), (\mathbf{x}_t', t'))\right)$$

- Modelling the dynamic function: spatial-temporal kernel

$$k\big((\mathbf{x}_t, t), (\mathbf{x}_t', t')\big) = \big(k_{\mathrm{S,Con}}(\mathbf{x}_t, \mathbf{x}_t') + k_{\mathrm{S,SE}}(\mathbf{x}_t, \mathbf{x}_t')\big) \cdot k_{\mathrm{T,Mat}}(t, t')$$

- **Kernel design**: Design spatial-temporal composite kernel function to account for the time-varying and non-stationary nature of the received signal strength map

III. Efficient BO with GP Factorisation

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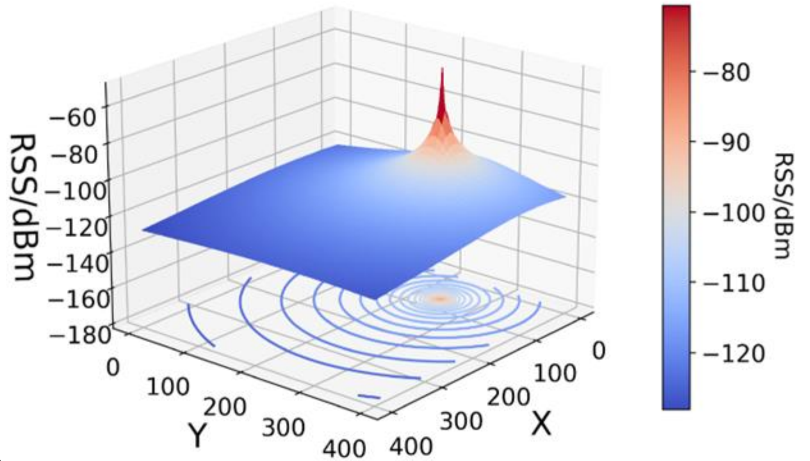
- **Kernel design**: Design spatial-temporal composite kernel function to account for the time-varying and non-stationary nature of the received signal strength map

$$k_{\mathrm{S,Con}}(\mathrm{x}_t, \mathrm{x}_t') = \Phi,$$

$$k_{\mathrm{S,SE}}(\mathbf{x}_t, \mathrm{x}_t') = \sigma_m^2 \exp\big(-\|\mathbf{x}_t - \mathbf{x}_t'\|^2 / l^2\big)$$

$$k_{\mathrm{T,Mat}}(t, t') = \sigma_m^2 \frac{2^{1-v}}{\Gamma(v)} \left(\frac{\sqrt{2v}\|t - t'\|}{l}\right)^v K_v\left(\frac{\sqrt{2v}\|t - t'\|}{l}\right)$$

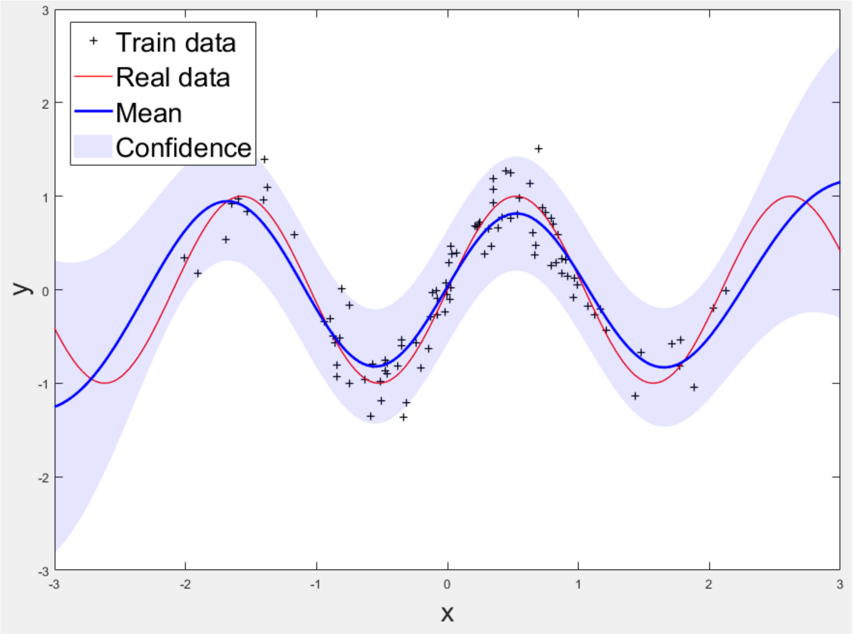


III. Efficient BO with GP Factorisation

Gaussian Process: Surrogate model of the unknown function

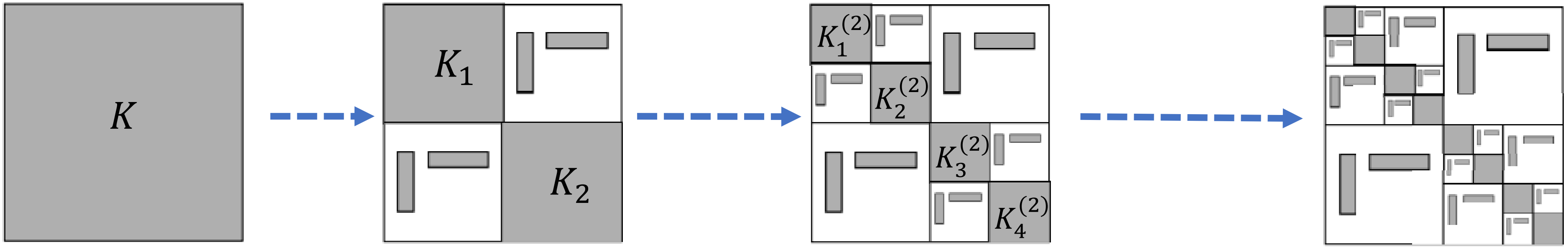
$$f(\mathbf{x}_t, t) \sim \mathcal{GP}\left(m(\mathbf{x}_t, t), k((\mathbf{x}_t, t), (\mathbf{x}'_t, t'))\right)$$

- The computational complexity is $O(n^3)$.
- Inducing points-based method
- Hierarchical off-diagonal low-rank (HODLR) factorisation method



III. Efficient BO with GP Factorisation

The dense covariance matrix can be hierarchically factored into a product of block low-rank updates of the the identity matrix. This is called the hierarchical off-diagonal low-rank (HODLR) factorisation.



$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_1 & \mathbf{U}_1\mathbf{V}_1^T \\ \mathbf{V}_1\mathbf{U}_1^T & \mathbf{K}_2 \end{bmatrix}$$

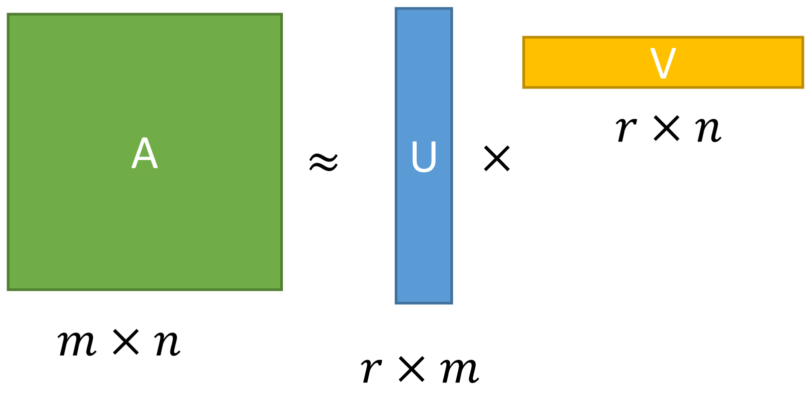
$$\mathbf{K}_1 = \begin{bmatrix} \mathbf{K}_1^{(2)} & \mathbf{U}_1^{(2)}\mathbf{V}_1^{(2)^T} \\ \mathbf{V}_1^{(2)}\mathbf{U}_1^{(2)^T} & \mathbf{K}_2^{(2)} \end{bmatrix}$$

$$\mathbf{K}_2 = \begin{bmatrix} \mathbf{K}_3^{(2)} & \mathbf{U}_2^{(2)}\mathbf{V}_2^{(2)^T} \\ \mathbf{V}_2^{(2)}\mathbf{U}_2^{(2)^T} & \mathbf{K}_4^{(2)} \end{bmatrix}$$

- The matrixes $\mathbf{K}$ are the dense parts.
- The off-diagonal blocks are compressed into the “tall” and “thin” U and V matrixes via low-rank approximation

III. Efficient BO with GP Factorisation

- Low-rank approximation would remain the main spectral features, which is efficient in the most off-diagonal covariance matrix. ($r \ll n$)



- Adaptive Cross Approximation decomposition O(rn).

- The computational complexity is O($rn \log^2 n + n \log n$).

Multi-agent BO

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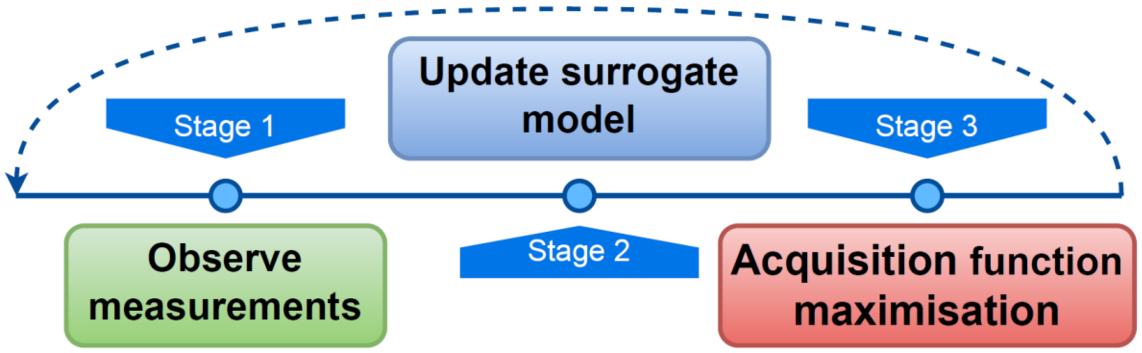


- In order to schedule multiple UAVs for search and tracking, the multi-point EI method is utilised to determine the measuring locations of UAVs sequentially:

$$EI_n(x_t^{1:q}, t^{1:q}) := \mathbb{E}[[\max_{i=1,\ldots,q} f(\mathbf{x}_t^i, t^i) - \tau_{n_t}]^+]$$

- Constant liar approximation is used to sequentially solve the multi-point EI

Algorithm 1 BO-assisted active sensing management

Require: Prior surrogate model $\mathcal{GP}_0$, initial data $\mathcal{D}_0$, UAV number K

while $t_i \leq T$ **do**

Stage 1:
- Receive the K RSS measurements
- Set the time stamp $t_i = \max\{t_i^1, t_i^2, \cdots, t_i^K\}$
- Augment data $\mathcal{D}_i \leftarrow \mathcal{D}_{i-1} \cup \{\mathbf{x}_{t_i}^k, t_i^k, y_{t_i}^k\}_{k=1}^K$

Stage 2:
- Update $\mathcal{GP}_i$

Stage 3:
- Set the start time stamp $t_s \leftarrow t_i + \psi$
- Update search bound of time scale as $\mathbf{t} = [t_s, t_s + \gamma]$
- Determine $\{\mathbf{x}_{t_{i+1}}^k\}_{k=1}^K$ and $\{t_{i+1}^k\}_{k=1}^K$ by sequentially maximising AF as follows:

$$\{\mathbf{x}_{t_{i+1}}^k, t_{i+1}^k\} = \arg\max_{\mathbf{x}_t \in \mathcal{X}, t \in \mathbf{t}} \alpha^k(\mathbf{x}_t, t)$$

- Send the UAVs to measure the RSSs at $\{\mathbf{x}_{t_{i+1}}^k, t_{i+1}\}_{k=1}^K$
- $i \leftarrow i + 1$

end while

IV. Numerical Results: Settings

- Log-distance path loss model:

$$y_{t_i} = y_{0,t_i} - \eta \log_{10}(d_{t_i}) + \epsilon_{t_i}, y_{0,t_i} = \text{-50dBm}, \forall t_i \in T, \eta = 3$$

- Area of interest: 400*400 m^2
- Target motion model: Constant velocity with initial state as $[50m, 1m/s, 50, 1m/s]$

- Benchmarks: 1) Proposed kernel used in GP with Cholesky factorisation; 2) Proposed kernel used in GP with HOLDR factorisation

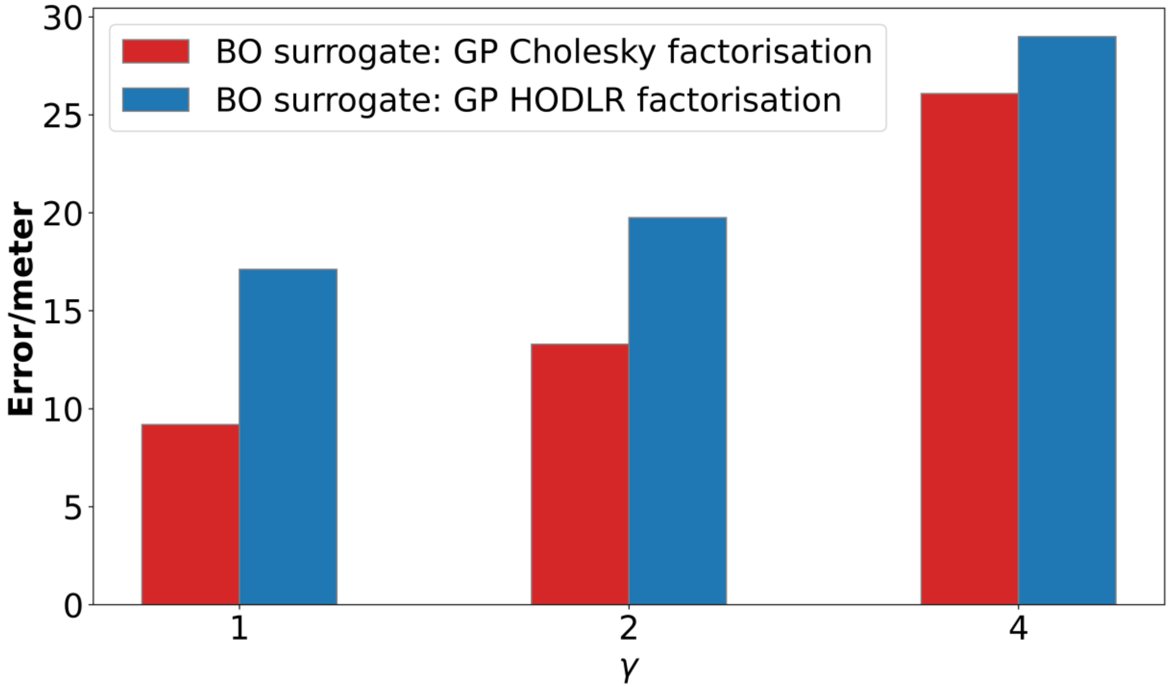
[1] F. M. Nyikosa, M. A. Osborne, and S. J. Roberts, “Bayesian optimization for dynamic problems,” arXiv preprint arXiv:1803.03432, 2018.

IV. Numerical Results: Running Time

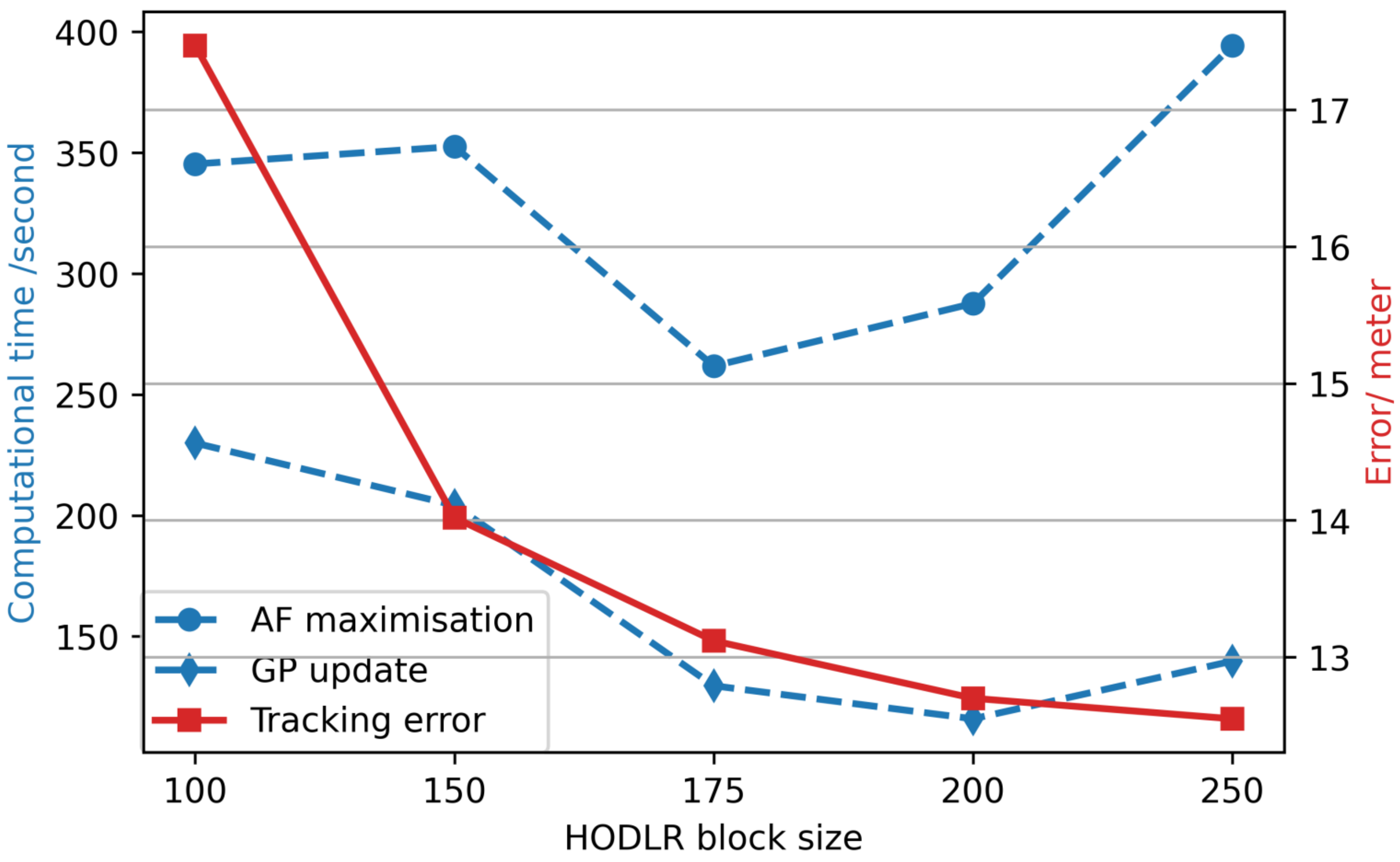
γ	Factorisation method	GP update (sec)	AF maximisation (sec)	
			1st UAV	2nd UAV
$\gamma = 1$	HODLR	0.86	0.61	0.62
	Cholesky	0.97	0.77	0.77
$\gamma = 2$	HODLR	0.29	0.72	0.73
	Cholesky	0.49	0.84	0.85
$\gamma = 4$	HODLR	0.04	0.79	0.80
	Cholesky	0.19	0.96	0.97

Per-step running time based on:

- GP with Cholesky factorisation
- GP with HODLR factorisation



IV. Numerical Results: Error



- ➢ HODLR factorisation helps to improve the efficiency of the proposed approach.

V. Conclusions and Future Plan

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- Conclusions

Efficient, factorized GP methods are developed for sensor scheduling and tracking in sensor networks

  - Sensor scheduling can be integrated into sensor networks for efficient sensor management

- Future plan

  - Improve the efficiency of Bayesian optimisation for sensor management and tracking: 1) Path planning for UAVs; 2) Error bound-assisted searching; 3) Extend the proposed approach to a heterogeneous sensor network case study

- Acknowledgements to UDRC SIGNeTs Project (funded by UK Dstl and USA DoD)

- Thanks!