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Consensus-based Distributed Variational Multi-object Tracker in Multi-Sensor Network

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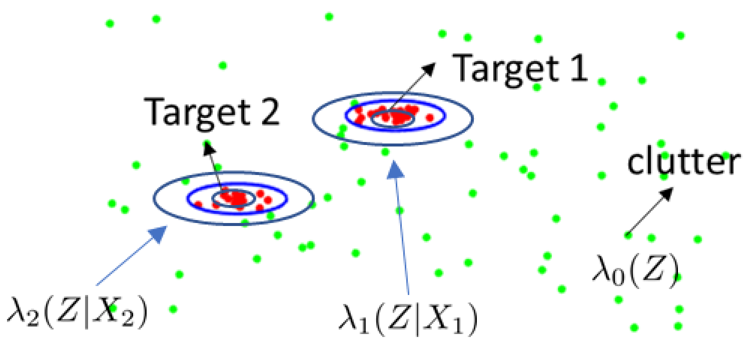
Outline

- Introduction
  - Scalable variational tracker: single sensor case
- Distributed fusion and tracking in multi-sensor network
  - Centralized Variational Tracker
  - Consensus-based distributed variational multi-object tracker
- Summary and future directions

Non-homogeneous Poisson process (NHPP) measurement model

Example: measurements of 2 targets and clutter process, generated by the NHPP model:



At time step n:	target state	X_n
	measurements	Z_n
	target number	K

Measurements from each object (i=1,…K) and clutter (i=0) follow a NHPP with intensity $\lambda_i(Z_n|X_{n,i})$

Total measurements follow an NHPP with intensity $\lambda(Z_n|X_n) = \sum_{i=0}^{K} \lambda_i(Z_n|X_{n,i})$

Likelihood function: $\frac{e^{-\sum_{i=0}^{K}\Lambda_i}}{M_n!} \prod_{j=1}^{M_n} \sum_{i=0}^{K} \lambda_i(Z_{n,j}|X_{n,i})$

Association prior $p(\theta_{n,j}|\Lambda) = \frac{\sum_{k=0}^{K} \Lambda_k \delta[\theta_{n,j} = k]}{\Lambda_s}$ categorical distribution

Data Association $\theta_n = [\theta_{n,1}, \ldots, \theta_{n,M_n}]$ $\theta_{n,j} = i$ $\begin{cases} i = 0 & \text{From clutter} \\ i \in \{1, \ldots, K\} & \text{From target i} \end{cases}$

[1] K. Gilholm, S. Godsill, S. Maskell, and D. Salmond, "Poisson models for extended target and group tracking," in Signal and Data Processing of Small Targets 2005, vol. 5913

Variational multi-object tracker: single sensor case

coordinate ascent variational filtering [2]:

target posterior distribution: $\hat{p}_n(X_n, \theta_n | Y_n)$

mean-field factorisation $q_n(X_n, \theta_n) = q_n(X_n) q_n(\theta_n)$

minimise the KL divergence

$\mathrm{KL}(q_n(X_n) q_n(\theta_n) || \hat{p}_n(X_n, \theta_n | Y_n))$

Implementation:

Iteratively update until convergence

1) update for $q_n(X_n)$

for $k = 1, 2, ..., K$ $q_n(X_{n,k}) = \mathcal{N}(X_{n,k}; \mu^k_{n|n}, \Sigma^k_{n|n})$, Kalman filter

2) update for $q_n(\theta_n)$

for $j = 1, ..., M_n$ $q_n(\theta_{n,j}) \propto \frac{\overline{\Lambda}_0}{V} \delta[\theta_{n,j} = 0] + \sum_{k=1}^{K} \overline{\Lambda}_k l_k \delta[\theta_{n,j} = k]$, categorical distribution

Scalable! All can be updated independently

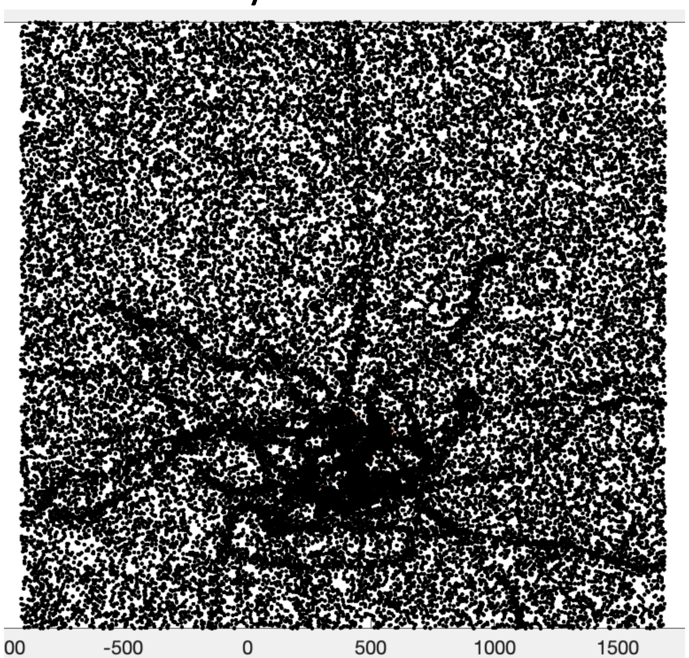
[2] Gan, R., Li, Q., & Godsill, S. (2022, July). A variational Bayes association-based multi-object tracker under the non-homogeneous Poisson measurement process. In 2022 25th International Conference on Information Fusion (FUSION) (pp. 1-8). IEEE.

Why we choose variational tracker

Tracking 20 targets with heavy clutter:

Target rate: 10

Clutter density: 10^{-4}



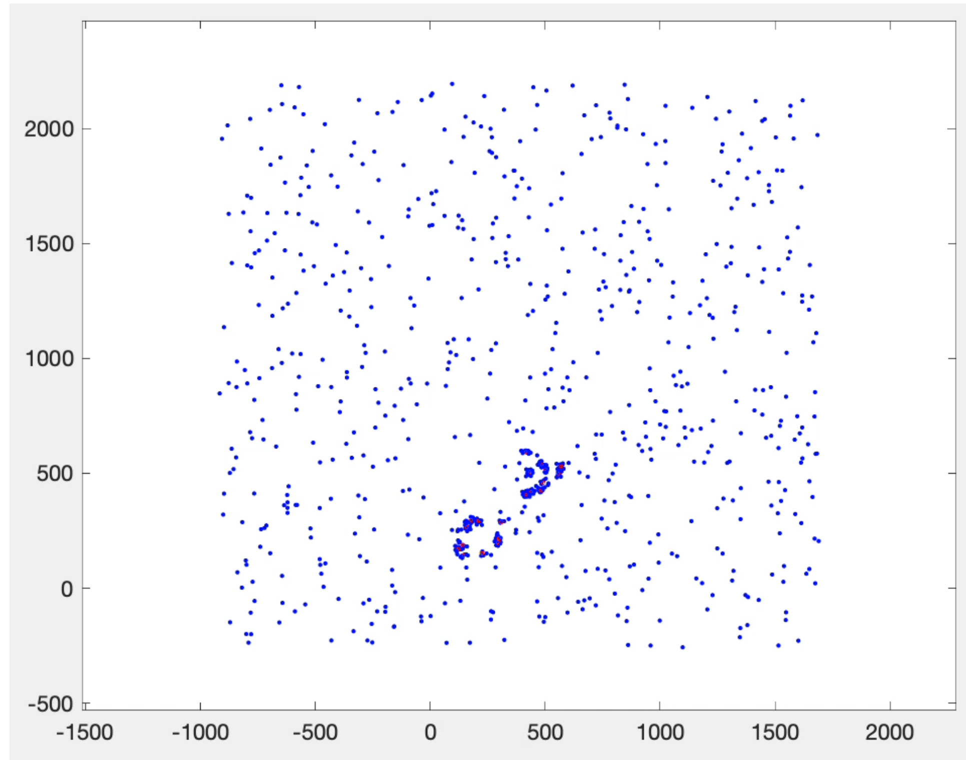


TABLE I: Tracking performance comparisons

Fast and accurate!

RMSE (mean $\pm 1\sigma$) | track loss percentage (%) | CPU time (s)

dataset	K	PF-NHPP			Gibbs-AbNHPP			ET-JPDA			VB-AbNHPP[(1)]			VB-AbNHPP		
1	2	7.69±0.64	0.00	4.05	5.50±0.34	0.00	0.22	7.49±1.06	4.50	5e-4	5.64±0.58	0.00	3e-6	5.51±0.43	0.00	6e-6
2	4	N/A	51.8	15.9	5.63±0.13	0.00	0.57	8.40±2.97	8.75	2e-3	5.91±0.37	0.25	6e-6	5.75±0.29	0.00	1e-5
3	10	–			6.06±0.25	0.10	0.67	9.41±1.99	16.3	0.01	6.50±0.50	2.20	8e-6	6.03±0.32	0.70	2e-5
4	20	–			6.25±0.26	0.65	2.31	N/A	22.6	0.03	7.31±0.70	4.05	1e-5	6.30±0.40	1.90	3e-5

Distributed tracking in multi-sensor network

Problem settings:

- A network of sensors tracking **a large number of targets** in clutter
- **Decentralized processing:**
  1) No central processing unit
  2) local communication with neighbours (constraints of bandwidth)
- Time-varying sensor network (communication link failure)

Defence Impacts: e.g., border surveillance, and maritime operations

- Fast and precise tracking
- Resilient for adversarial disruptions and communication constraints

Proposed methods:

Variational trackers in multi-sensor network

- Centralised fusion
- Optimal distributed fusion based on consensus algorithm
- Comparison: suboptimal distributed fusion

Aim to achieve an equivalent result

fuse local posteriors using arithmetic average

Measurement and association model for a sensor network

Consider a sensor network with N_s sensors

For each sensor *s* (*s*=1,2,…, N_s) :

- **local measurements $\boldsymbol{Y_n^s}$:**
  independent NHPP model with Poisson rate Λ^s
- **Likelihood function:**

$$p(Y_n^s|\theta_n^s, X_n) = \prod_{j=1}^{M_n^s} \ell^s(Y_{n,j}^s|X_{n,\theta_{n,j}^s}),$$

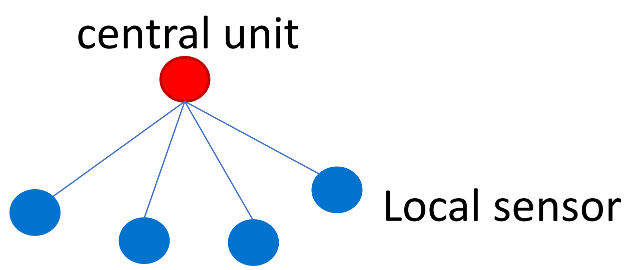
- **Association prior**

$$p(\theta_{n,j}^s) = \frac{\sum_{k=0}^{K} \Lambda_k^s \delta[\theta_{n,j}^s = k]}{\Lambda_{sum}^s}$$

Categorical distribution

For all N_s sensors at the central unit:

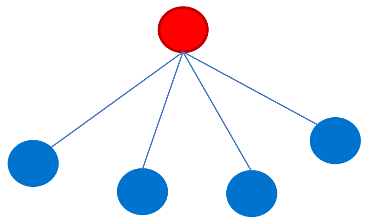
- **joint likelihood** $p(Y_n|\theta_n, X_n) = \prod_{s=1}^{N_s} p(Y_n^s|\theta_n^s, X_n).$
- **joint association prior** $p(\theta_n|M_n) = \prod_{s=1}^{N_s} p(\theta_n^s|M_n^s),$



Centralized variational multi-object tracker

Coordinate ascent update: $q_n(X_n, \theta_n) = q_n(X_n) q_n(\theta_n)$

Iteratively update until convergence



1. Update for $q_n(X_n)$

$$q_n(X_n) \propto \hat{p}_n(X_n) \prod_{k=1}^{K} \mathcal{N}(\overline{Y}_n^k; H X_{n,k}, \overline{R}_n^k)$$ Kalman filter

$$\overline{Y}_n^k = \overline{R}_n^k \sum_{s=1}^{N_s} \Omega_{k,2}^s, \qquad \Omega_{k,2}^s = R_k^{s\,-1} \sum_{j=1}^{M_n} q_n(\theta_{n,j}^s = k) Y_{n,j}^s.$$

$$\overline{R}_n^k = \left(\sum_{s=1}^{N_s} \Omega_{k,1}^s \right)^{-1} \qquad \Omega_{k,1}^s = R_k^{s\,-1} \sum_{j=1}^{M_n} q_n(\theta_{n,j}^s = k),$$

Independently update for each target k

- prediction $\hat{p}_n(X_{n,k}) = \mathcal{N}(X_{n,k}; \mu_{n|n-1}^{k*}, \Sigma_{n|n-1}^{k*}).$
- Kalman filter update $q_n(X_{n,k}) = \mathcal{N}(X_{n,k}; \mu_{n|n}^k, \Sigma_{n|n}^k)$

2. Update for $q_n(\theta_n)$

$$q_n(\theta_n) = \prod_{s=1}^{N_s} q_n(\theta_n^s).$$

Independently update for each sensor s, each association j

$$q_n(\theta_{n,j}^s) \propto \frac{\Lambda_0^s}{V^s} \delta[\theta_{n,j}^s = 0] + \sum_{k=1}^{K} \Lambda_k^s l_k^s \delta[\theta_{n,j}^s = k].$$ Categorical distribution

How to decentralise it ?

Coordinate ascent update: $q_n(X_n, \theta_n) = q_n(X_n) q_n(\theta_n)$

Iteratively update until convergence

1. Update for $q_n(X_n)$ $\quad q_n(X_n) \propto \hat{p}_n(X_n) \prod_{k=1}^{K} \mathcal{N}\left(\overline{Y}_n^k; HX_{n,k}, \overline{R}_n^k\right)$

$$\Omega_{k,2}^s = R_k^{s\,-1} \sum_{j=1}^{M_n} q_n(\theta_{n,j}^s = k) Y_{n,j}^s.$$

$$\Omega_{k,1}^s = R_k^{s\,-1} \sum_{j=1}^{M_n} q_n(\theta_{n,j}^s = k),$$

Compute at each sensor

$$\overline{Y}_n^k = \overline{R}_n^k \sum_{s=1}^{N_s} \Omega_{k,2}^s,$$

$$\overline{R}_n^k = \left(\sum_{s=1}^{N_s} \Omega_{k,1}^s \right)^{-1}$$

Consensus:

When converge, each sensor has:

$$\frac{1}{N_s} \sum_{s=1}^{N_s} \Omega_{k,1}^s$$

$$\frac{1}{N_s} \sum_{s=1}^{N_s} \Omega_{k,2}^s.$$

Independently update for each target k

- Kalman filter update $\quad q_n(X_{n,k}) = \mathcal{N}(X_{n,k}; \mu_{n|n}^k, \Sigma_{n|n}^k)$

2. Update for $q_n(\theta_n)$

Independently update for each sensor s, each association j

$$q_n(\theta_{n,j}^s) \propto \frac{\Lambda_0^s}{V^s} \delta[\theta_{n,j}^s = 0] + \sum_{k=1}^{K} \Lambda_k^s l_k^s \delta[\theta_{n,j}^s = k].$$

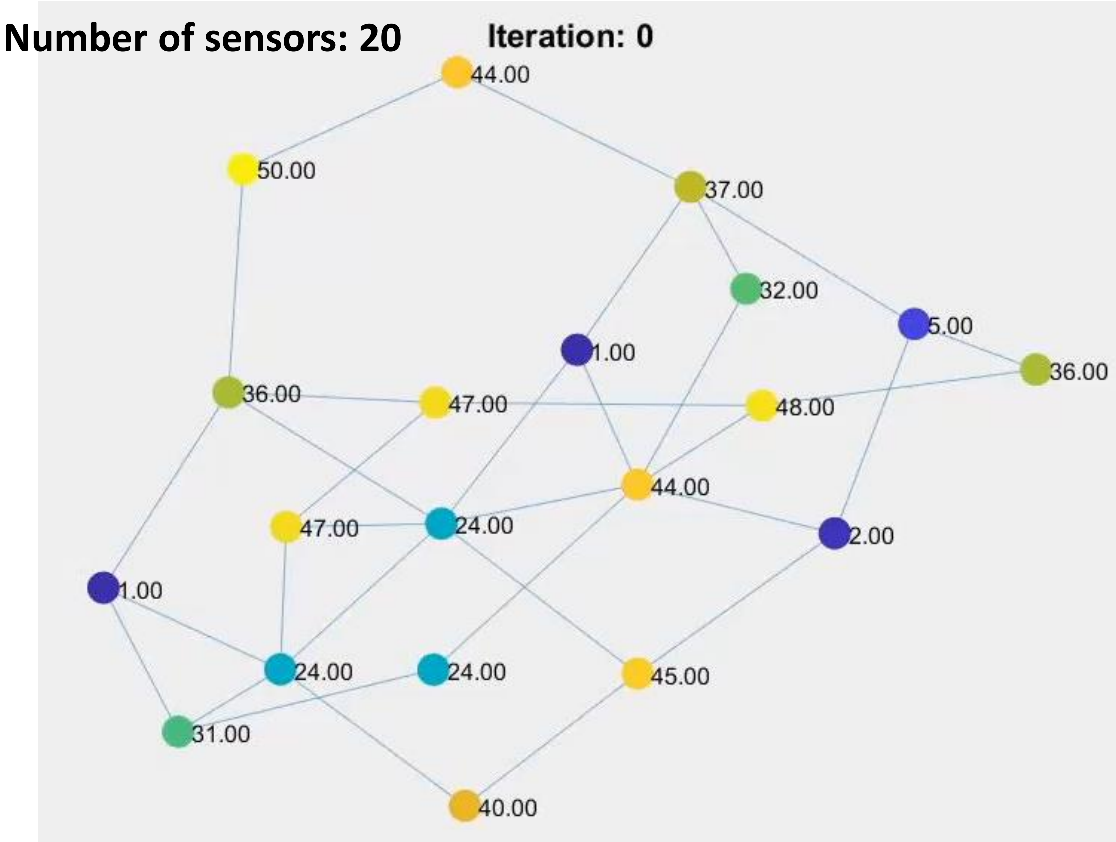
A demo of average consensus algorithm

- Initial iteration: Every node starts with an initial value $\Omega_{k,1}^{s}$.
- For each iteration:

  each node communicates with its neighbors to collect their values, and update value :

$$\hat{\Omega}_{k,1}^{(s,m+1)} = W_{ss}^{(m)}\hat{\Omega}_{k,1}^{(s,m)} + \sum_{j\in\mathcal{N}_s(m)} W_{sj}^{(m)}\hat{\Omega}_{k,1}^{(j,m)}$$

- Output: When converge, each node should have same value $\frac{1}{N_s}\sum_{s=1}^{N_s}\Omega_{k,1}^{s}$

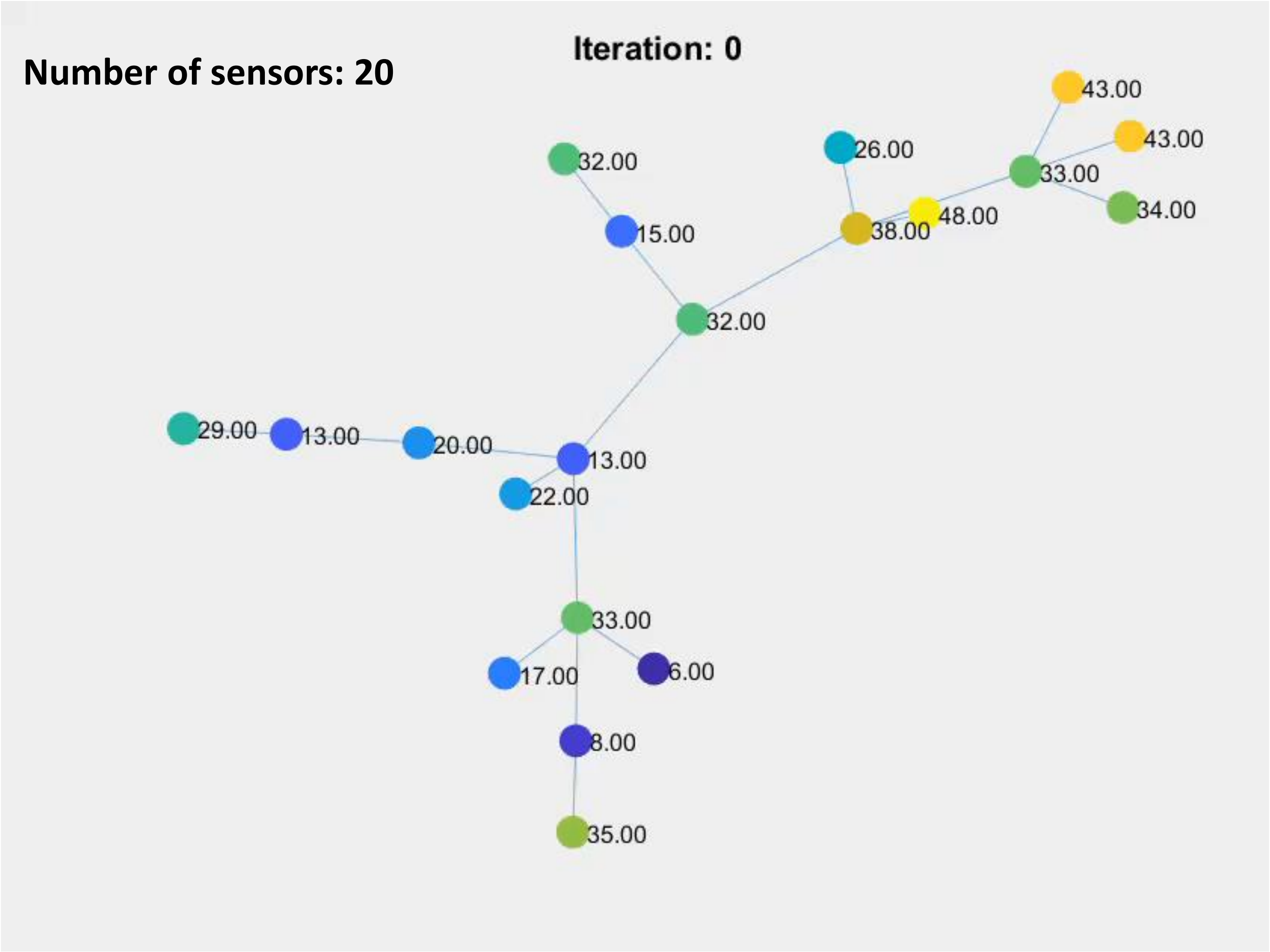


Initial value: range from 1-50

Output: average value 31.66

Works in time-varying sensor network!

- Each sensor node has $\Omega_{k,1}^{s}$ locally
- When converge, each sensor has the same value $\frac{1}{N_s}\sum_{s=1}^{N_s}\Omega_{k,1}^{s}$



Consensus-based distributed variational multi-object tracker

Objective: with only local communications,
each sensor has the same estimate as the centralized sensor fusion that have all data

Implementation: At each sensor $s = 1,2, \dots, N_s$

1. Update for $q_n(X_n)$

   Step 1. compute $\Omega_{k,1}^s$, $\Omega_{k,2}^s$ locally

   Step 2. perform **average consensus** to get
   $$\hat{\Omega}_{k,1} = \frac{1}{N_s} \sum_{s=1}^{N_s} \Omega_{k,1}^s$$
   $$\hat{\Omega}_{k,2} = \frac{1}{N_s} \sum_{s=1}^{N_s} \Omega_{k,2}^s.$$

   Step 3. for each target k

   Kalman filter update $q_n(X_{n,k}) = \mathcal{N}(X_{n,k}; \mu_{n|n}^k, \Sigma_{n|n}^k)$

   using $\overline{Y}_n^k = \overline{R}_n^k (N_s \hat{\Omega}_{k,2})$ $\overline{R}_n^k = (N_s \hat{\Omega}_{k,1})^{-1}$

2. Update for $q_n(\theta_n)$

   for each association j $q_n(\theta_{n,j}^s) \propto \frac{\Lambda_0^s}{V^s} \delta[\theta_{n,j}^s = 0] + \sum_{k=1}^{K} \Lambda_k^s l_k^s \delta[\theta_{n,j}^s = k].$

Results

Settings:

Number of sensors: 20

Number of targets: 50

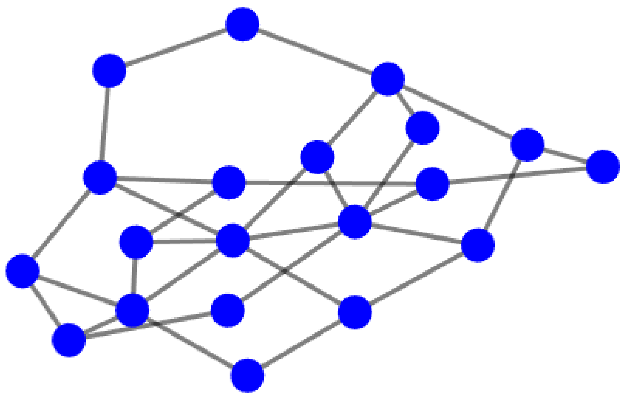
Target rate:1

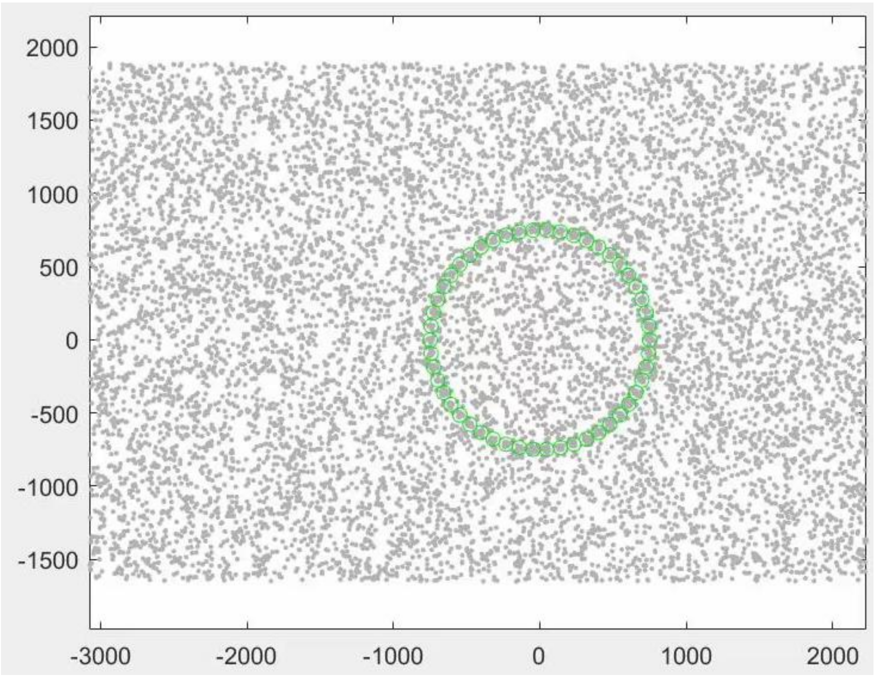
Clutter rate: 500

Measurements from all sensors (grey dots)

Ground truth tracks (black lines)

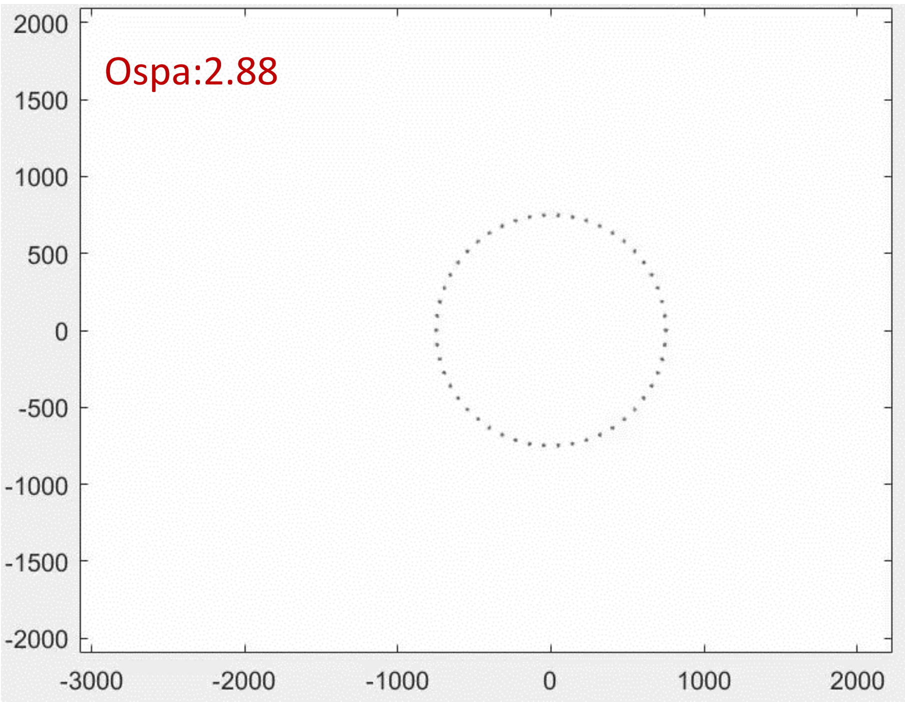
Target initial positions (green circles)



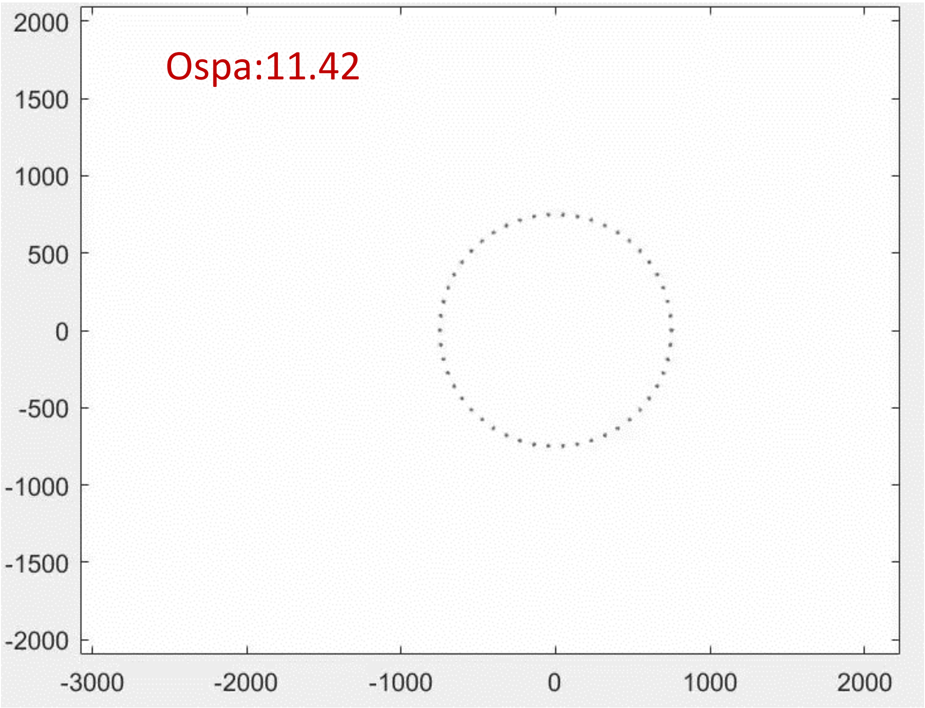


Results-an example from one single run

Optimal distributed fusion



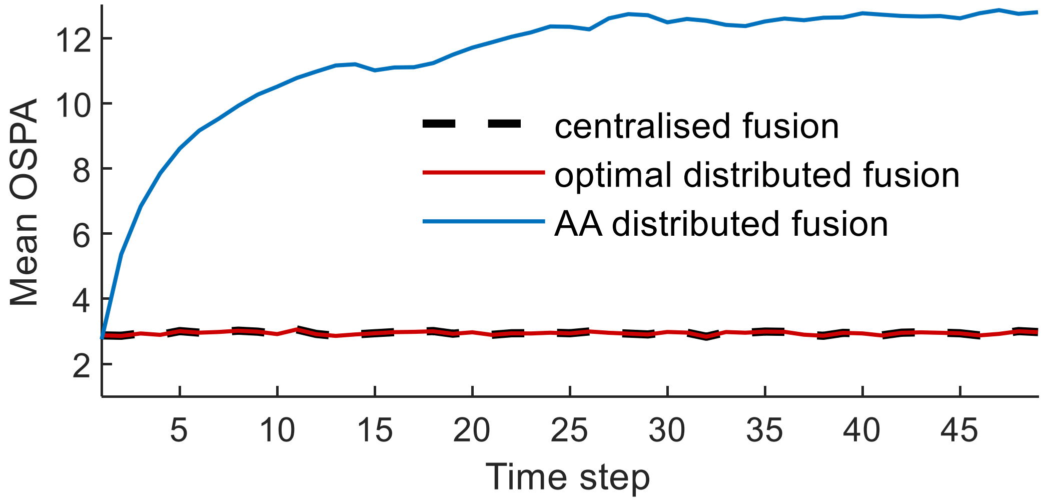
Suboptimal arithmetic average distributed fusion



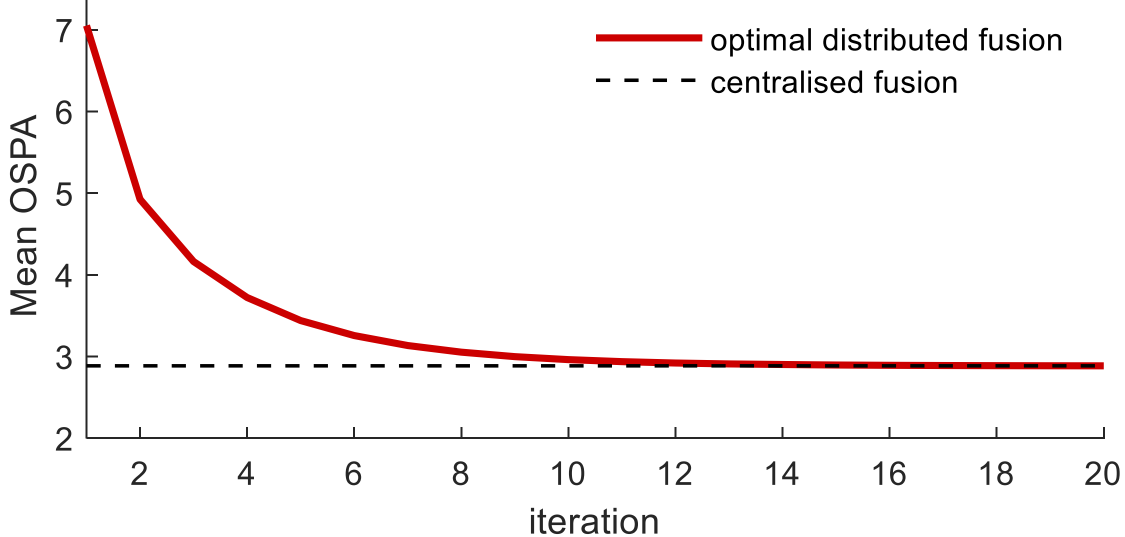
Ground truth tracks: black lines
Estimate position: dotted colored line
95% confidence ellipse: shaded circles

Results

Mean OSPA of different fusion methods (average consensus iteration is 20)



Mean OSPA of the optimal distributed fusion over different iterations



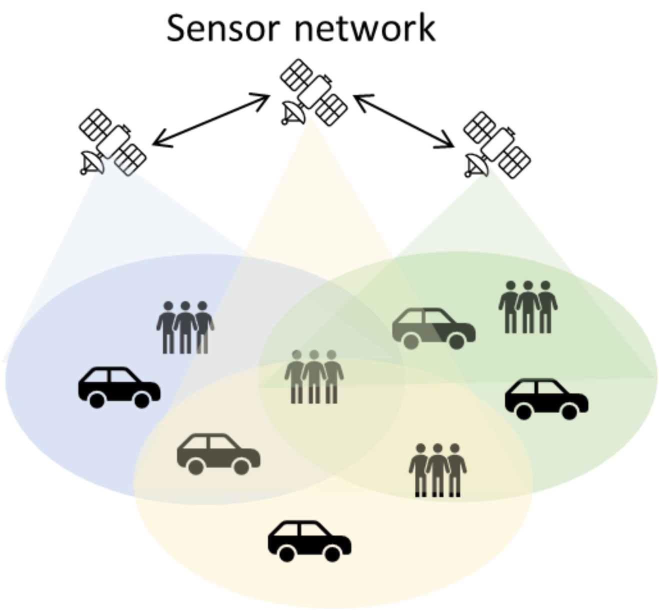
Summary and extensions

Summary: A consensus-based distributed variational tracker

1. **Scalable to target and measurement number**
2. **Achieve optimal fusion with distributed implementation**
3. **Reliable solution: work in time-varying communication links**

Extensions

1. **A more flexible scheme that allows each sensor operate independently without waiting for consensus;**
2. **Solutions for a more general heterogeneous sensor network**
   - sensor network with different coverage
   - measurement function can be nonlinear (range, bearing)
3. **More robust and versatile tracking**
   - Variational tracker with missed objects relocation for heavy clutter cases [3]



[3] Gan, R., Li, Q. and Godsill, S., “Variational Bayes Tracker with Non-homogeneous Poisson Measurement Process”, IEEE Transactions on Aerospace and Electronic Systems, 2023, submitted.