Instabilities in Navigation Balancing on the Head of a Pin

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Instabilities in Navigation
Balancing on the Head of a Pin

James Davies, Lee Devlin, Angel Garcia-Fernandes, Elias Griffith, Marcel Hernandez, Paul Horridge, Simon Maskell, Alexey Narykov, Alex Phillips, Christian Pollitt, Chris Taylor, Alessandro Varsi, Murat Uney, Michael Wright, and many others

Signal Processing Group, UoL

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Outline

- Inertial Sensing
  - Sensors and Measurements
- Dead Reckoning with an INS
  - Approximations
  - Coriolis Effects and Gravity
- Strapdown INS
  - Error Growth & Instabilities
- Augmentation
  - Position Fixing Methods – Map-Matching
- Applications
  - Land, Air and Sea
- Quantum Sensing
  - Atom interferometry as a Gravity Sensor
- Gravity Based Map-Matching
  - Gravity Maps and Gravity Gradient Sensors

Inertial Sensing

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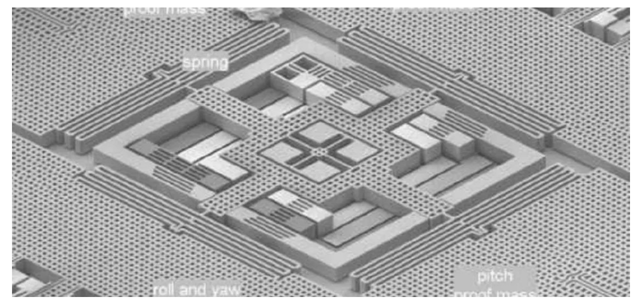
Inertial Sensors



Gyroscopes

Accelerometers

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https://www.canalgeomatics.com/product/kvh-dsp-3000-fiber-optic-gyro/

https://www.tekedia.com/what-is-an-accelerometer-what-is-a-gyroscope/

Inertial Navigation Systems

Strapdown Inertial Navigation System

https://www.unmannedsystemstechnology.com/expo/inertial-navigation-systems/

Inertial Sensing

- Accelerometers measure ‘specific force’ along a specific axis,

$$\underline{f} = \underline{a} - \underline{g}$$

  - Accelerometers rely on measuring the deflection of a ‘proof mass’, either the physical motion or some quantity derived from a deflection, such as a vibrational frequency

- Gyroscopes measure angle rate around a specific axis,

$$\underline{\omega} = \frac{d\underline{\theta}}{dt}$$

  - Gyroscopes can either be optical (using the Sagnac effect) or mechanical (generally using opposing accelerometers)

Dead Reckoning with an INS

A Simple Approach to Inertial Navigation

- To calculate the velocity and position, first remove gravity from the measurements

$$\underline{a}(t) = \underline{\hat{f}}(t) + \underline{g}$$

  - And integrate

$$\underline{v}(t) = \underline{v}(0) + \int (\underline{\hat{f}}(t) + \underline{g})dt$$

  - And integrate again

$$\underline{r} = \underline{r}(0) + \int \underline{v}(t)\,dt$$

  - But you need to know where you are, and where you are heading…

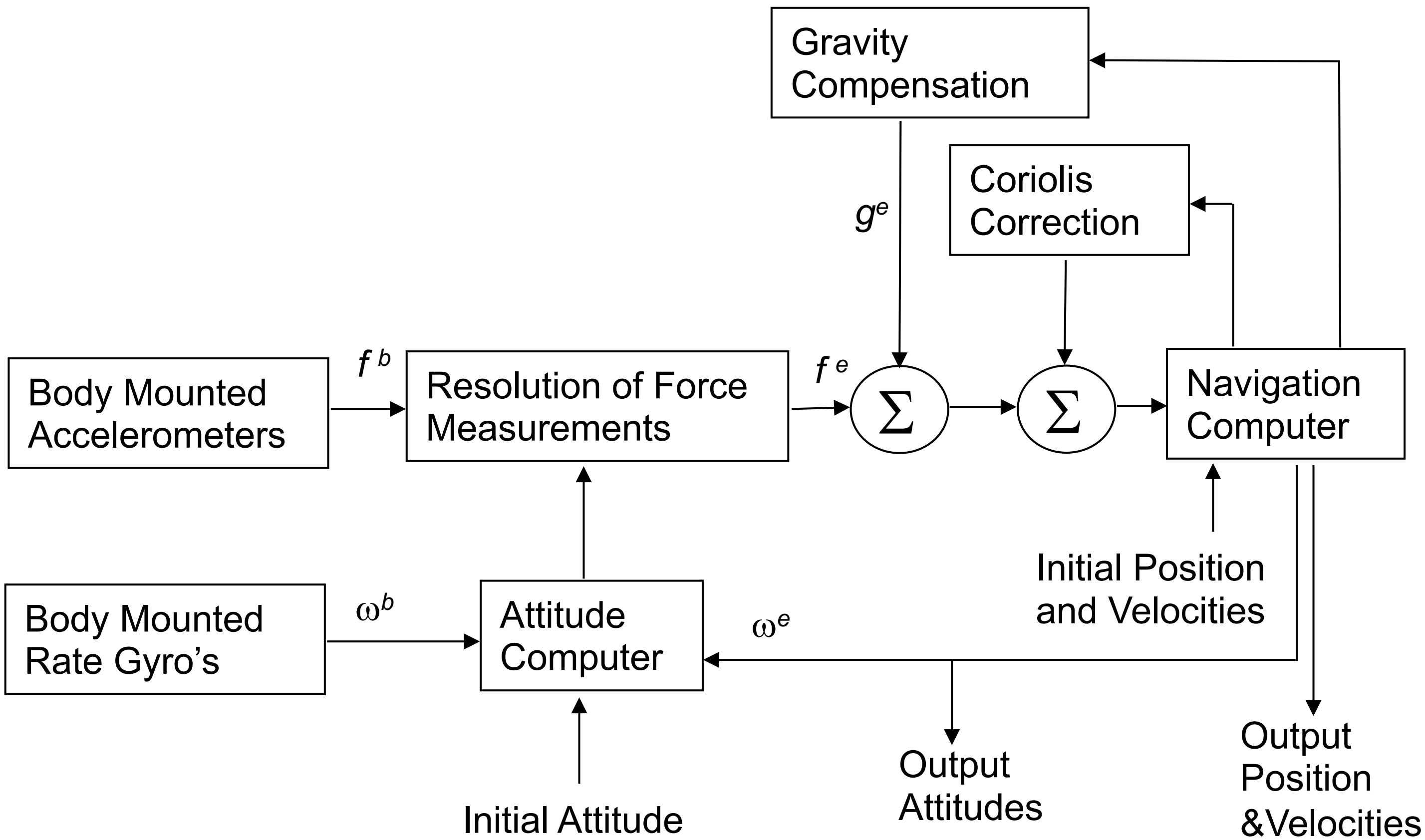
- That's without knowing which way is up, which way is North, and the gravity value at every point along your route

Biases and other errors

- Sensors have errors
  - Measurement noise, bias errors, alignment errors…
- Inertial sensors measure a derivative or the second derivative of the quantities that we want – position, velocity, orientation
  - Integrating does not correct to remove initial errors or any integrated errors
  - Integrating acceleration does not give velocity, it is the change in velocity
- Typically, we have to contend with:
  - Measurement errors: accumulate $\propto \sqrt{time}$
  - Bias errors: accumulate $\propto time$
  - Alignment errors: cause cross-coupling between errors
  - Scaling errors: accumulate, proportional to the signal
  - Time-dependent errors: bias drifts
  - Mechanical errors: caused by flexure of structure with rapid changes

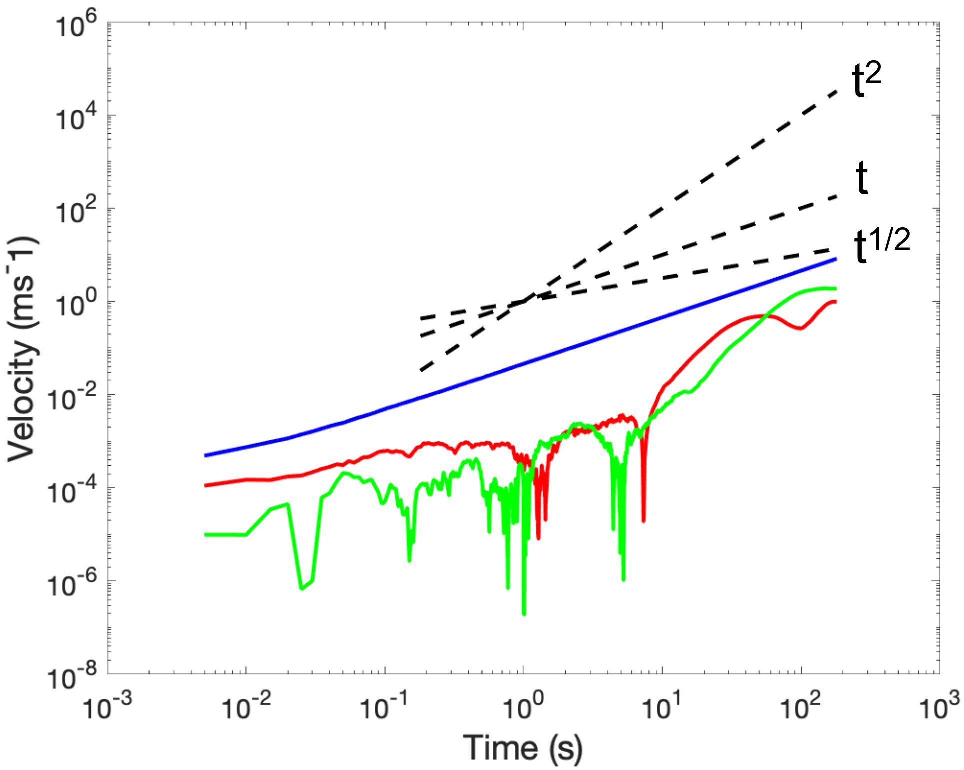
Strapdown INS

Strapdown Inertial Navigation System



Error Growth in Inertial Navigation Systems

- Example commercial MEMS INS using data from a static test



Log-Log plot of Velocity errors for static data test (x axis – red, y axis – green, z axis – blue). The black dashed lines are present to indicate functions that scale as square root of time, linear in time, and the square of time.

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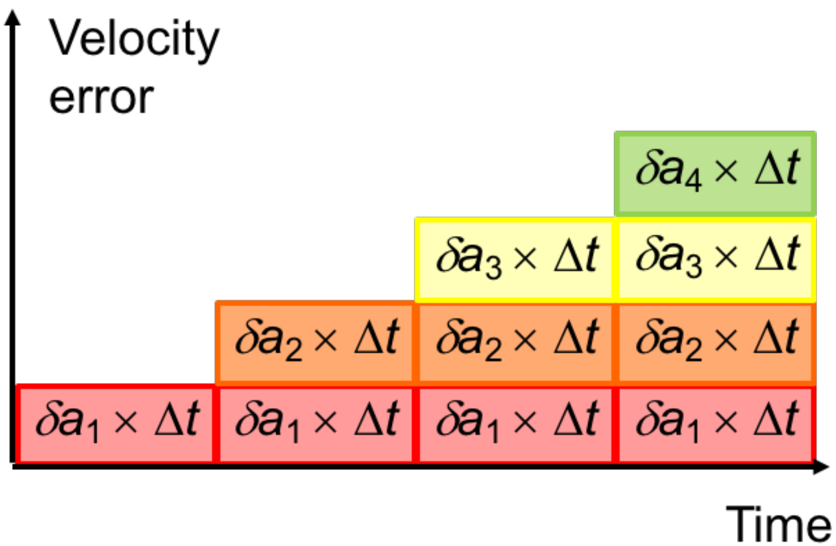
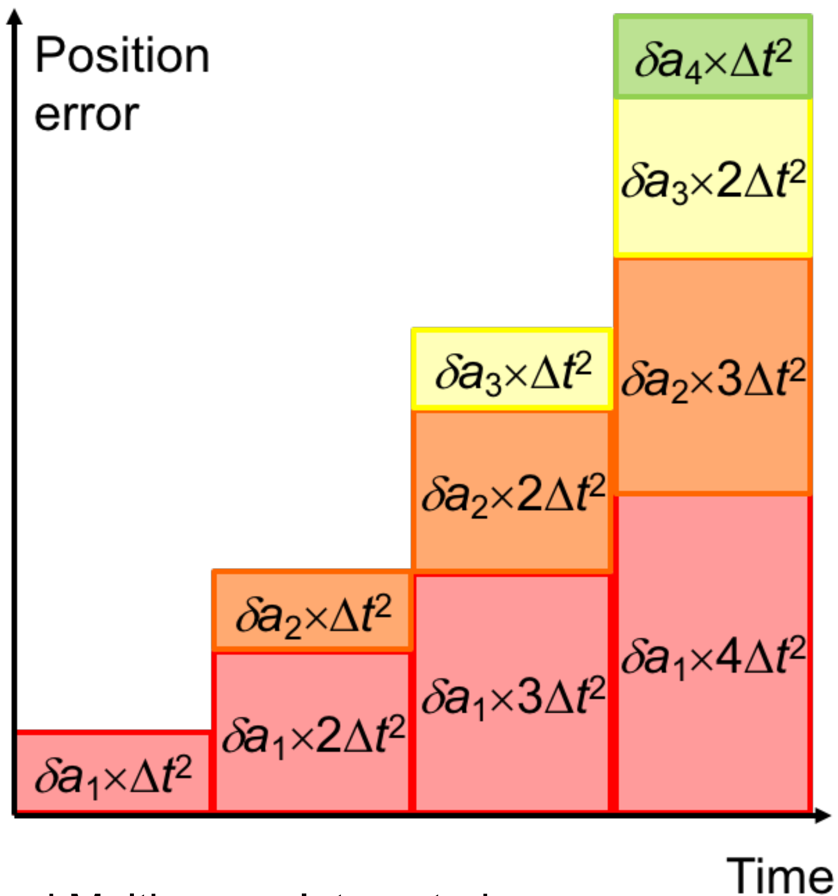
Accumulation of Errors





*Courtesy of Paul Groves (UCL)
See: P. D. Groves, “Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems” (Artech House, 2013).

Augmentation

Augmentation

- Augmentation – Improving Inertial Measurement Models
  - Dynamical models
  - Improved gravity models
  - Transfer alignment
  - Calibration
  - Magnetic sensors

- In practice, any INS needs a **position fixing system** to correct for the errors that accumulate over time.
  - GNSS/GPS (inc. signal information/phase)
    Loosely coupled, tightly coupled, ultra-tightly coupled, ‘deep’ coupled…
  - TRN/SMAC
  - Map-matching
  - Star trackers
  - Radio Navigation (eLORAN, VOR/DME/NDB, Opportunistic)

Error Propagation vs Bandwidth

- Dead Reckoning
  - Direct integration of inertial sensor outputs can provide high bandwidth measurements and rapid updates for location, velocity and attitude
  - Outputs from high bandwidth sensors can be integrated into control systems
  - Autonomous operation
  - Integration of velocity, acceleration and angle rates do not give direct measurements for position and attitude, just changes – **errors accumulate leading to instabilities and unconstrained drift**

- Position Fixing
  - Provide **direct** measurements for position which limits any accumulation of errors or drift
  - Non-autonomous, requires external reference or database
  - Low bandwidth updates due to strong correlations in databases and references over short periods of time
  - Overheads associated with maintenance of databases or reference signals

Quantum Sensing

Augmentation

- Augmentation – Improving Inertial Measurements
  - Dynamical models
  - Transfer Alignment
  - Calibration
  - Magnetic Sensors

- In practice, any INS needs a **position fixing system** to correct for the errors that accumulate over time
  - GNSS/GPS (inc. signal information/phase)

    Loosely coupled, tightly coupled, ultra-tightly coupled, 'deep' coupled…
  - TRN/SMAC
  - Map-matching
  - Star trackers
  - Radio Navigation (eLORAN, VOR/DME/NDB, Opportunistic)

Augmentation **with Quantum Sensors**

- Augmentation – Improving Inertial Measurements
  - Dynamical models
  - Transfer Alignment
  - **Calibration – As a minimum, quantum sensors can dramatically improve the calibration of existing inertial sensors***
  - Magnetic Sensors

- In practice, any INS needs a **position fixing system** to correct for the errors that accumulate over time
  - GNSS/GPS (inc. signal information/phase)

    Loosely coupled, tightly coupled, ultra-tightly coupled, 'deep' coupled…
  - TRN/SMAC
  - **Gravity Map Matching****
  - Star trackers
  - Radio Navigation (eLORAN, VOR/DME/NDB, Opportunistic)

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* MJ Wright, et al. "Cold atom inertial sensors for navigation applications." *Frontiers in Physics* 10 (2022): 994459.
** AM Phillips, et al. "Augmented inertial navigation using cold atom sensing." Cold Atoms for Quantum Technologies 11578 (2020): 115780C

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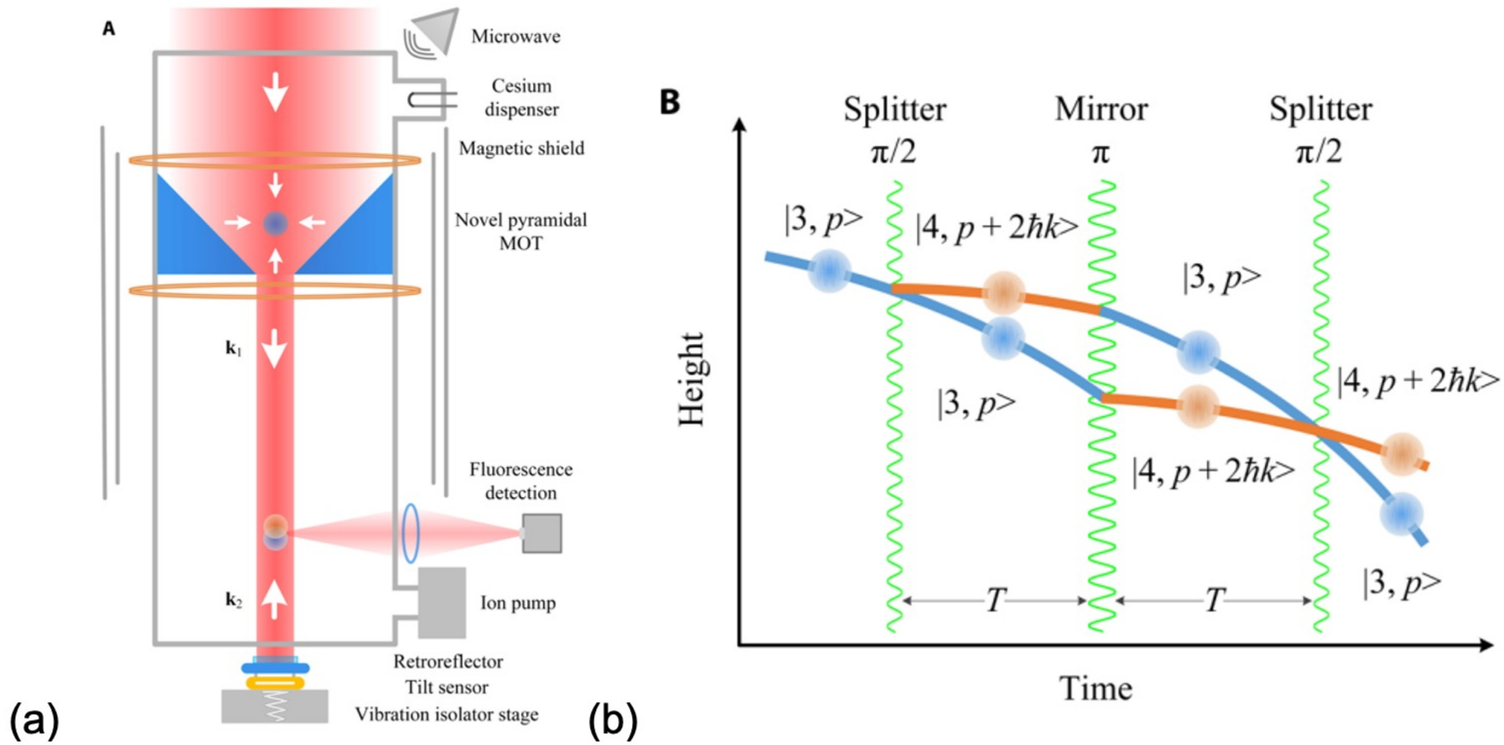
Cold Atom Sensors

- One of the biggest sources of systematic errors in navigation systems is inaccurate calibration of the sensors
  - Even good calibration will inevitably lead to systematic errors
- Accelerometers rely on the ability to measure the motion of a proof mass when it undergoes an acceleration
  - The mass of a proof mass is only known to some finite accuracy
- Atoms of one isotope are all identical, and their mass is known to an extremely high precision
- Atoms behave like waves, ‘matter waves’, and (when sufficiently cold) a cloud of atoms of the same type, can be made to form superpositions and generate interference patterns
  - This allows very accurate phase measurements to be made which can be used to estimate the motional states of the atoms

Cold Atom Interferometers



Schematic diagrams showing (a) an example of the geometry of a cold atom interferometer and (b) the pulse sequence used to generate superpositions

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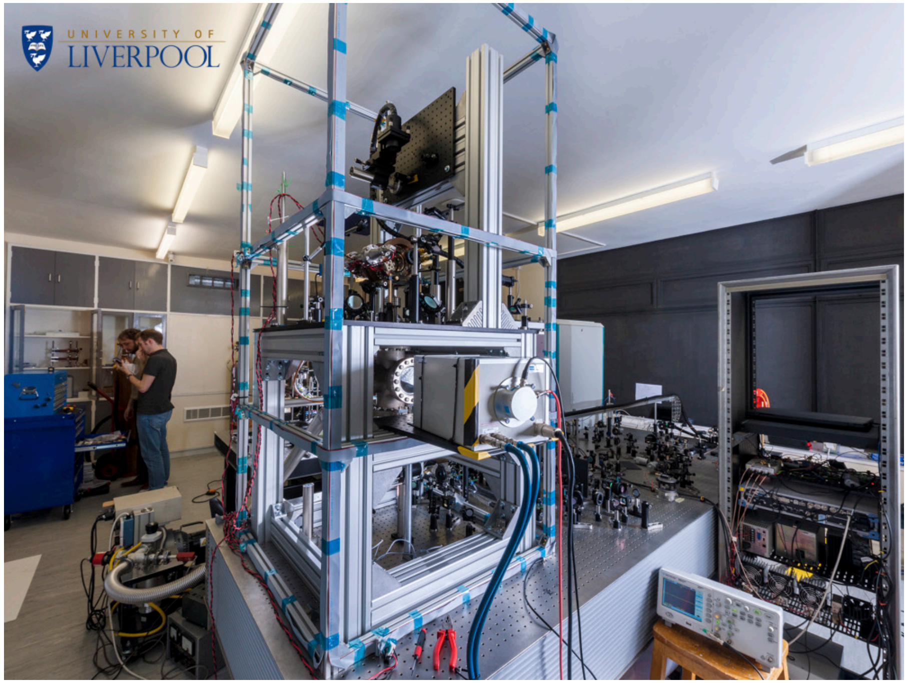
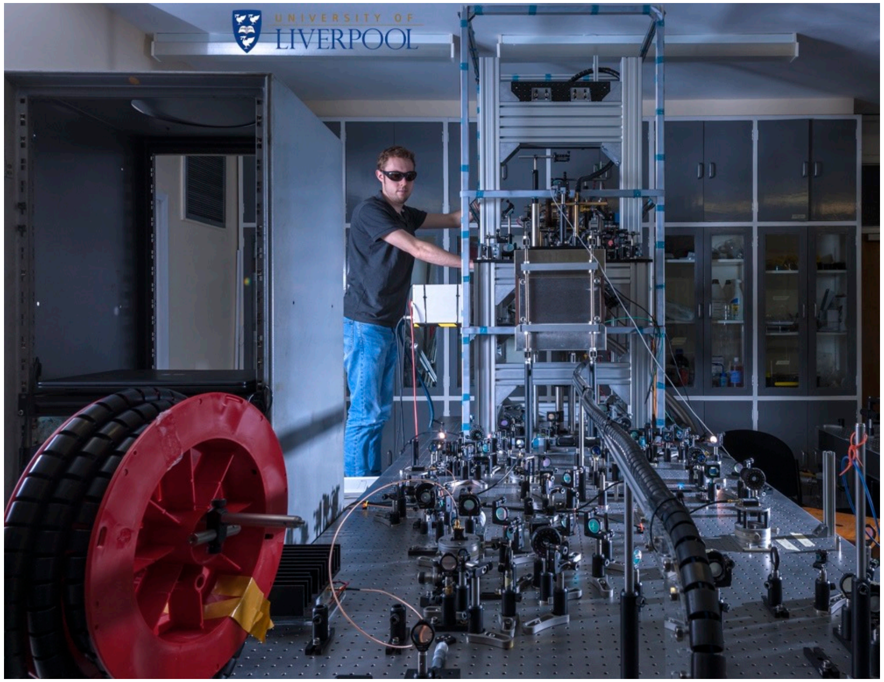
Wu, Xuejian, Zachary Pagel, Bola S. Malek, Timothy H. Nguyen, Fei Zi, Daniel S. Scheirer, and Holger Müller. “Gravity surveys using a mobile atom interferometer.” Science advances 5, no. 9 (2019): eaax0800.

Atom Interferometry

- Quantum Inertial Sensors
  - User-defined duty cycle
  - Low measurement frequency
  - Possible time-dependent sensitivity
  - Restricted dynamic range
  - Measurement failure rate
  - Orientation limits





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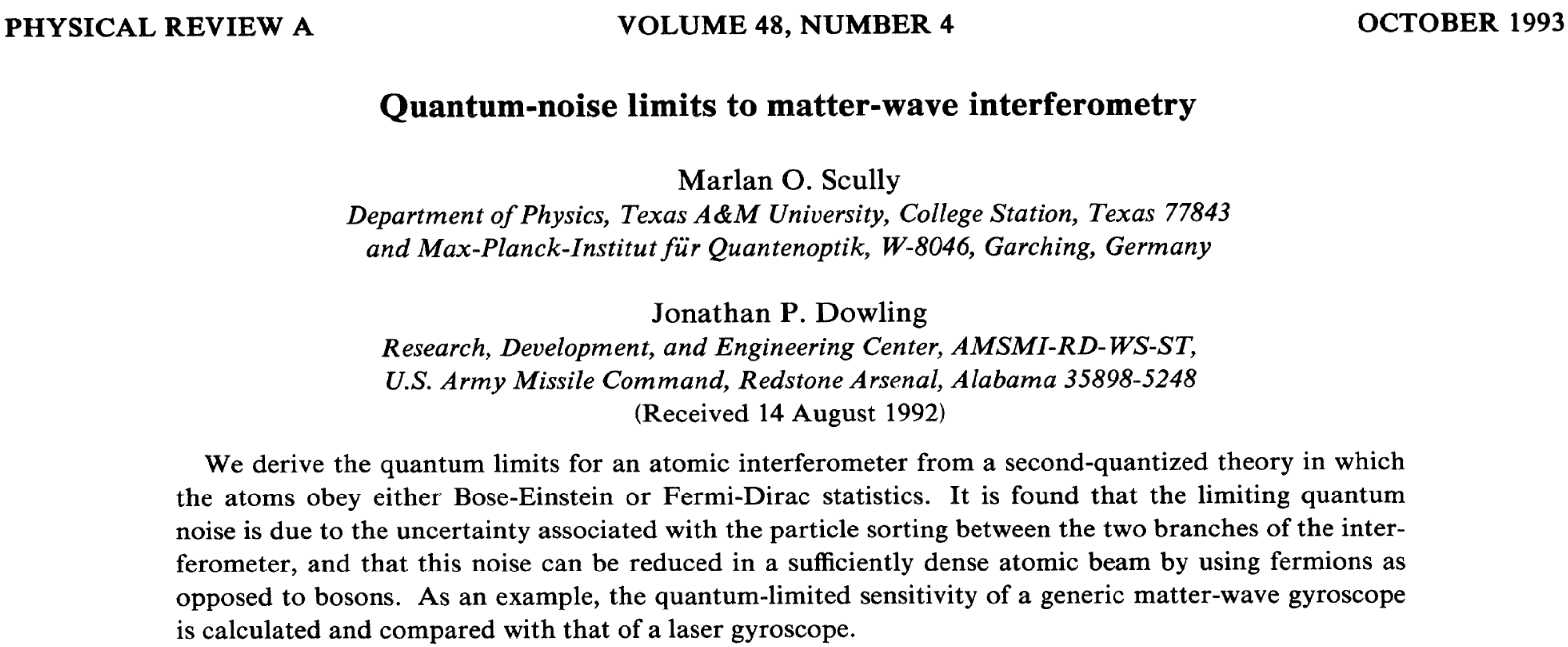
So, why is it so good?


PHYSICAL REVIEW A VOLUME 48, NUMBER 4 OCTOBER 1993

Quantum-noise limits to matter-wave interferometry

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and Max-Planck-Institut für Quantenoptik, W-8046, Garching, Germany

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U.S. Army Missile Command, Redstone Arsenal, Alabama 35898-5248
(Received 14 August 1992)

We derive the quantum limits for an atomic interferometer from a second-quantized theory in which the atoms obey either Bose-Einstein or Fermi-Dirac statistics. It is found that the limiting quantum noise is due to the uncertainty associated with the particle sorting between the two branches of the interferometer, and that this noise can be reduced in a sufficiently dense atomic beam by using fermions as opposed to bosons. As an example, the quantum-limited sensitivity of a generic matter-wave gyroscope is calculated and compared with that of a laser gyroscope.

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So, why is it so good?

TABLE I. Compared and contrasted are different properties of matter-wave and optical gyroscopes in terms of their sensitivity to phase differences—or equivalently—rotation rates. We see that the high mass of atoms initially contributes an increase of sensitivity of 10^{10}, but that the low atomic beam intensity, compared to photon beams, removes some of this advantage, as does the reduced number of round trips possible in an atom interferometer. Nevertheless, a typical factor of a 10^4 increase in rotation sensitivity can still be expected using atoms rather than photons.

	Matter	Laser	Matter-to-light sensitivity factor
Mass factor	$\sim 10^4$ MeV	~ 1 eV	$\sim 10^{10}$
Flux	$\rho v A \sim 10^{10} \times 10^4 \times 10^{-2}$ $= 10^{12} \dfrac{\text{particles}}{\text{sec}}$	$\dfrac{P}{\hbar v} \sim \dfrac{10^{-3}}{10^{-19}}$ $= 10^{16} \dfrac{\text{photons}}{\text{sec}}$	$\sim 10^{-2}$
Round trips	~ 1	$\sim 10^4$	$\sim 10^{-4}$

So, why is it so good?

PHYSICAL REVIEW A VOLUME 57, NUMBER 6 JUNE 1998

Correlated input-port, matter-wave interferometer: Quantum-noise limits to the atom-laser gyroscope

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(Received 15 September 1997; revised manuscript received 9 February 1998)

TABLE I. Compared and contrasted are different properties of one- and two-port matter-wave and optical gyroscopes in the terms of their sensitivity to phase differences—or equivalently—rotation rates. We see that the high mass of atoms initially contributes an increase of sensitivity of 10^{10}, but that the low atomic beam intensity, compared to photon beams, removes some of this advantage, as does the reduced number of round-trips possible in an atom interferometer.

	Matter	Laser	One-port atom-to-light factor	Two-port matter-to-light factor	Two-port to one-port atom	Two-port to one-port light	Two-port atom to one-port light
Mass factor	10^4 MeV	1 eV	10^{10}	10^{10}	1	1	10^{10}
Flux	$N=10^{12}\,\frac{\text{particles}}{\text{sec}}$	$N=10^{16}\,\frac{\text{photons}}{\text{sec}}$	$\sqrt{10^{12}/10^{16}}=10^{-2}$	$\frac{10^{12}}{10^{16}}=10^{-4}$	$\frac{10^{12}}{\sqrt{10^{12}}}=10^{6}$	$\frac{10^{16}}{\sqrt{10^{16}}}=10^{8}$	$\frac{10^{12}}{\sqrt{10^{16}}}=10^{4}$
Round-trips	1	10^4	10^{-4}	10^{-4}	1	1	10^{-4}

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Cold Atoms as Sensors

- The falling cloud of cold atoms is placed in a superposition, which is then swapped over, and recombined – this gives an interference pattern
  - The phase of the interference pattern is proportional to the gravity

$$\Delta\phi_0 = k_{eff} g(z_0) T^2$$

  - where k_{eff} is the wavenumber of the pi/2 pulses.
- For two interferometers, sharing a common phase reference signal (the main Raman beam), the phase difference is

$$\begin{aligned}\Delta(\Delta\phi) &= \Delta\phi_0 - \Delta\phi_1 \\ &= k_{eff}(g(z_0) - g(z_1))T^2 \\ &\simeq k_{eff}\Delta z(dg_z/dz)T^2\end{aligned}$$

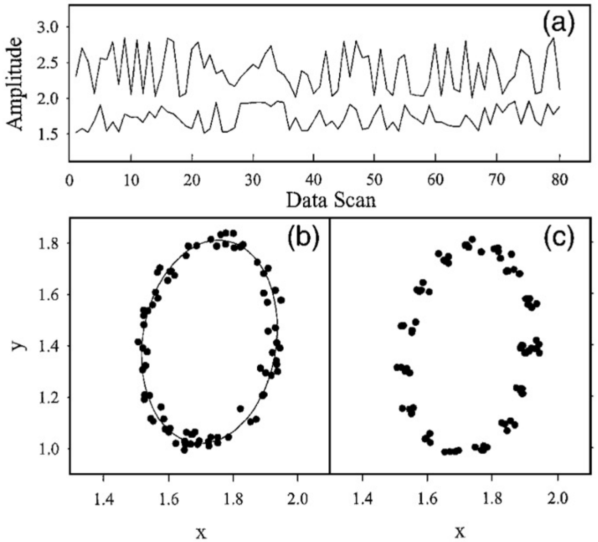
  - Technically, there is a rotation term in there as well, but this is proportional to the square of the rotation rate, so as long as $\Omega^2 \ll dg_z/dz$ then the gravity gradient will dominate the measurement

Gravity Gradiometry

Measurements

$$S_0(n) = \eta(\bar{N} + \delta N_{n,0})\sin(\Delta\phi_0 + \phi_n) + s_0$$

$$S_1(n) = \eta(\bar{N} + \delta N_{n,1})\sin(\Delta\phi_1 + \phi_n + \delta\phi_n) + s_1$$



Example gravity gradient ellipse** where the amplitudes in (a) show the two signals S_0 and S_1.

System parameter	Parameter Value
Type of atoms	Caesium 133
Mass of atom	2.20693925 x 10^{-25} kg
Duty Cycle	0.32
Measurement frequency	1 Hz
Measurement efficiency, η	0.5
Number of atoms in cloud, N	10^6
Shot noise, $\sigma_N = \sqrt{N}$	10^3
Time between pulses, T	160 ms
Effective wave number, k_{eff}	1.4649 x 10^7 m^{-1}
Horizontal Raman beamwidth	1 cm
Vertical separation of sensors, Δz	0.5 m
Phase noise, $\sigma_{\Delta\varphi}$	$\leq$ 20 mrad
Gravity gradient sensitivity (ellipse)	$\leq$ 7 x 10^{-8} s^{-2}

Example sensor parameter values

** Foster, G. T., J. B. Fixler, J. M. McGuirk, and M. A. Kasevich. "Method of phase extraction between coupled atom interferometers using ellipse-specific fitting." Optics letters 27, no. 11 (2002): 951-953.

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Gravity Map-Matching

Gravity Map-Matching

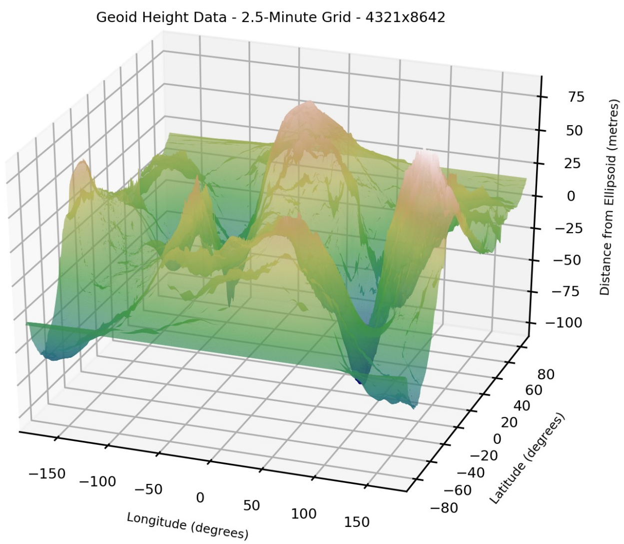
- The gravitational structure of the Earth at a large scale is complex, but it is also well studied
- Standard global gravity databases exist and are freely available
  - For example, the EGM2008 gravity model is a global database with a resolution of 1 nautical mile
  - More detailed databases do exist: e.g. SRTM2gravity model has provided a gravity model that has a minimum resolution of 90 metres, based on 'forward modelling' inferring gravity from the local topology

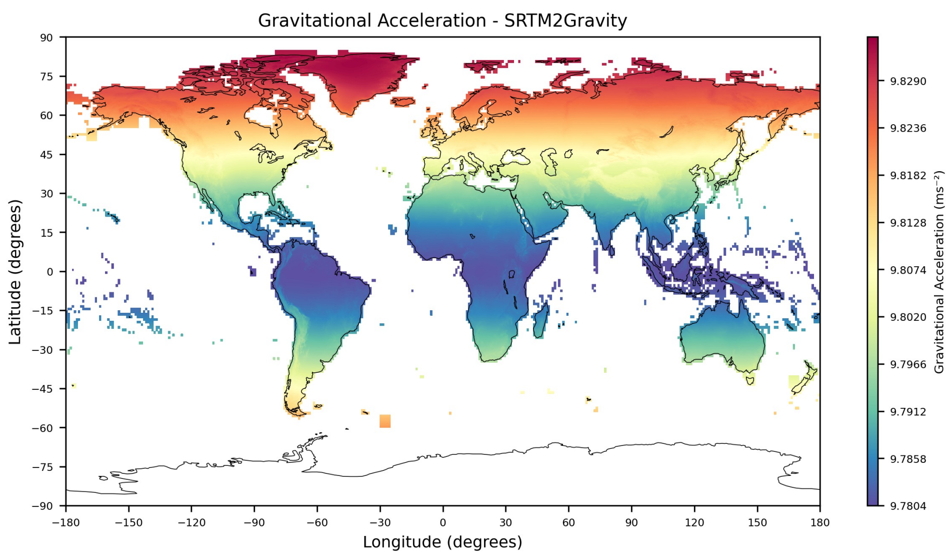
- Measuring the gravity gradient and comparing the measured values against the known gravity gradient values near to the estimated location of the platform allows corrections to be applied to stabilise the navigation solution
  - The method proposed here is based on particle filters using the characteristic measurements from a paired cold atom interferometer

AM Phillips, et al. "Position fixing with cold atom gravity gradiometers." *AVS Quantum Science* 4, no. 2 (2022)
JM Davies et al., "Navigating with Quantum Sensors using Gravity Gradients", NATO SET 311 Proc., Paper B55 (2023).

Gravity Reference Databases

- Somigliana at the WGS84 ellipsoid surface extrapolated for altitude
  - Uniform field
- Earth Gravitational Model EGM2008*: WGS84 version
  - Global, measured, 1 nautical mile resolution
- STRM2gravity** generated by forward modelling of topology
  - Nearly global, over land, modelled rather than measured, 90m resolution





* N. K. Pavlis et al., J. Geophys. Res. 117, B04406, https://doi.org/10.1029/2011JB008916 (2012).
** C. Hirt et al., Geophys. Res. Lett. 46, 4618, https://doi.org/10.1029/2019GL082521 (2019).

Particle Filters

- Particle filters are a type of Bayesian estimation method referred to as Sequential Monte Carlo (SMC) methods*,**
- Particle filters take sampled points in state space and use these as candidate solutions
  - The candidate solutions / particles have a weight (probability), and this weight is updated every time a measurement is taken
  - The closer the actual measurement is to the measurement predicted by the candidate solution / particle, the higher weight it attracts
- After a while, the weight is concentrated on the 'good' particles
  - To stop the weight being too focused on a few particles, the particles are resampled to produce more solutions close to the 'good' particles
- Iterating this process gradually selects the solutions that best match the measurements that are taken

* M. Arulampalam et al., IEEE Trans. Signal Process. 50, 174 (2002).
** O. Cappe et al., Proc. IEEE 95, 899 (2007).

Modelling and Simulation

- We use standard models for the inertial navigation systems
  - based on: P. D. Groves, Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems, 2nd ed. (Artech House, Boston, MA, 2013)
  - The INS is assumed to be a standard aviation or maritime grade INS
- The position fixing is done by estimating a correction vector to the navigation vector

TABLE I. Error values used for the INS sensor simulations.

Sensor error	Error value (1 std dev.)
Accelerometer static bias	30 μg
Accelerometer non-orthogonality	10 μrad
Accelerometer scaling error	10 ppm
Accelerometer measurement noise	15 $\mu g/\sqrt{\mathrm{Hz}}$
Gyroscope static bias	0.05 μrad
Gyroscope non-orthogonality	10 μrad
Gyroscope scaling error	10 ppm
Gyroscope measurement noise	2.0 $\mu\mathrm{rad/s}\ \sqrt{\mathrm{Hz}}$

$$\underline{X}_t = \begin{pmatrix} \Phi \\ \Lambda \\ h \\ u \\ v \\ w \\ a_x \\ a_y \\ a_z \\ \psi \\ \theta \\ \phi \\ P \\ Q \\ R \end{pmatrix} = \begin{pmatrix} \text{latitude,}^\circ \\ \text{longitude,}^\circ \\ \text{altitude, m} \\ x \text{ velocity, body axes, m/s} \\ y \text{ velocity, body axes, m/s} \\ z \text{ velocity, body axes, m/s} \\ x \text{ acceleration, body axes, m/s}^2 \\ y \text{ acceleration, body axes, m/s}^2 \\ z \text{ acceleration, body axes, m/s}^2 \\ \text{platform heading,}^\circ \\ \text{platform pitch,}^\circ \\ \text{platform roll,}^\circ \\ \text{angle rate, body } x \text{ axis,}^\circ\text{/s} \\ \text{angle rate, body } y \text{ axis,}^\circ\text{/s} \\ \text{angle rate, body } z \text{ axis,}^\circ\text{/s} \end{pmatrix}.$$

Correction of Inertial Drift

- We use a correction vector to continually correct our navigation solution

$$\Delta \underline{X}_t^{(NED)} = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta u \\ \Delta v \\ \Delta w \\ \Delta \psi \\ \Delta \theta \\ \Delta \phi \end{pmatrix} = \begin{pmatrix} \text{North correction, NED, metres} \\ \text{East correction, NED, metres} \\ x \text{ velocity correction, body, m/s} \\ y \text{ velocity correction, body, m/s} \\ z \text{ velocity correction, body, m/s} \\ \text{platform heading correction, deg} \\ \text{platform pitch correction, deg} \\ \text{platform roll correction, deg} \end{pmatrix}$$

- The particle filter uses $N_p = 500$ particles, each with a weight $w_t^{(i)}$
- The reweighting is done with a Gaussian update

$$\tilde{w}_t^{(i)} = \exp\left(- \frac{(\min |\underline{S}_{meas} - \underline{S}^{(i)}|)^2}{2\sigma_S^2} \right) w_{t-\Delta t}^{(i)}$$

  - where $\sigma_S \simeq \sqrt{\sigma_N^2/\bar{N}^2 + \sigma_\phi^2} = \sqrt{1/\bar{N} + \sigma_\phi^2}$ and weights are renormalized after the update

$$\sum_{i=1}^{N_p} w_t^{(i)} = 1$$

- And resampling is done when $N_{eff} = 1/(\sum_i (w_t^{(i)})^2) < N_p/2$

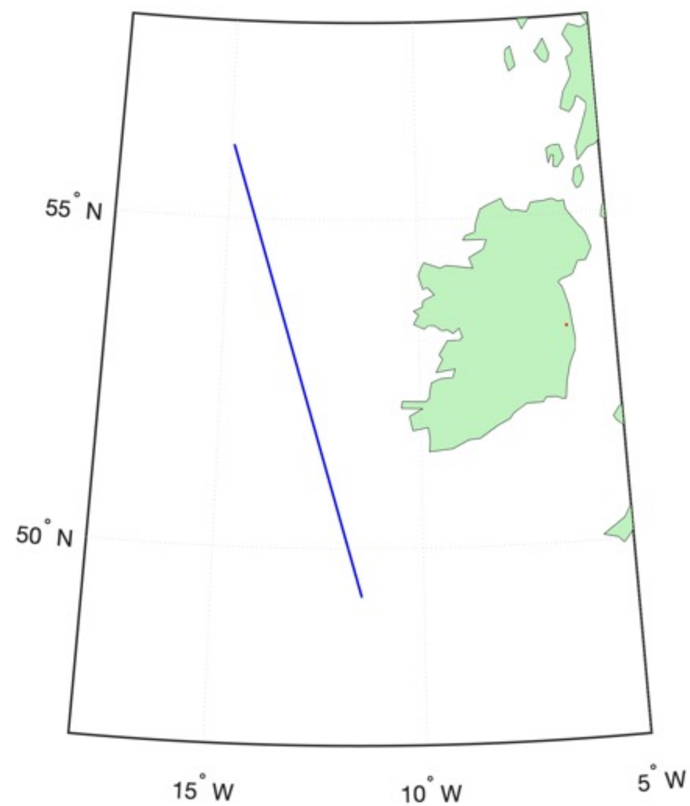
Example – Maritime Trajectory

- North Atlantic, off the West Coast of the island of Ireland
  - Bottom right to top left

- Approx 800km
- Altitude, -100m
- Speed, 10 m/s
- Duration, 22 hour (simulated)

- Holonomic constraints on motion

- Sensor Parameters based on published results from **University of Birmingham**



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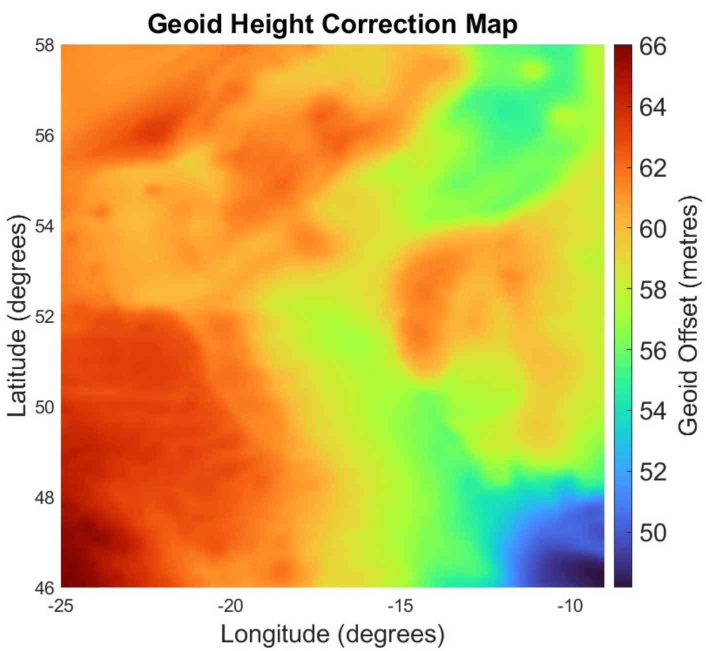
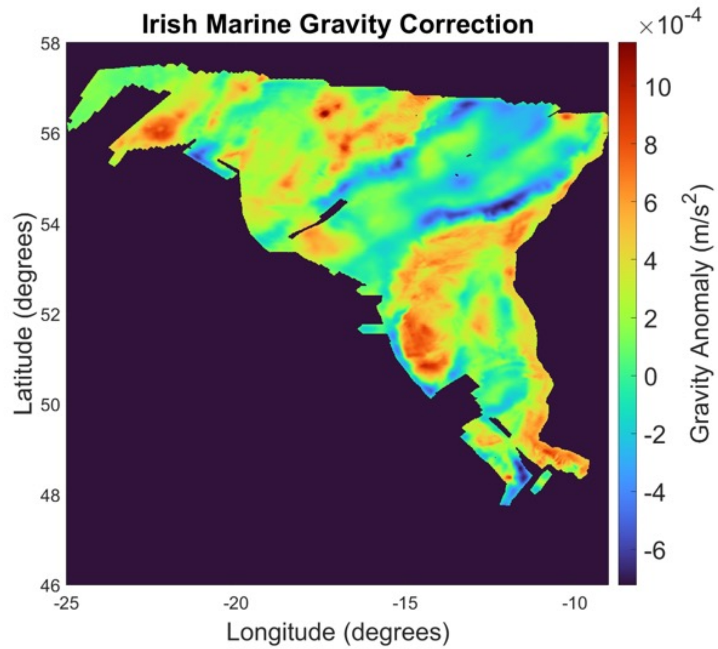
Example – North-West Irish Atlantic Coast





Section of EGM2008 global gravity database and the Irish sea data used for the scenario examined in this work

Pavlis, Nikolaos K., Simon A. Holmes, Steve C. Kenyon, and John K. Factor. "The development and evaluation of the Earth Gravitational Model 2008 (EGM2008)." *Journal of geophysical research: solid earth* 117, no. B4 (2012).

INFOMAR, https://www.infomar.ie/data [accessed Jan 2022].

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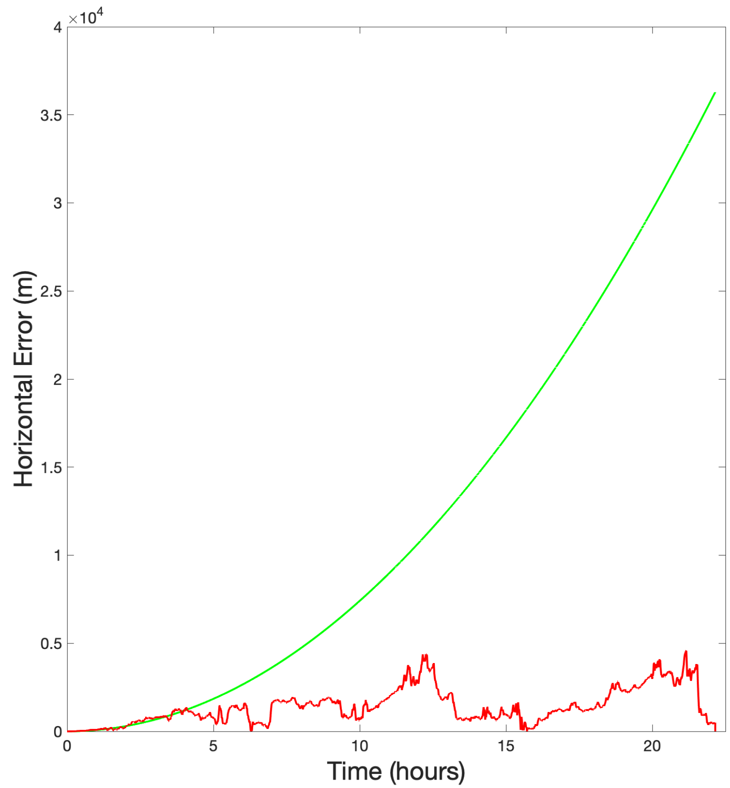
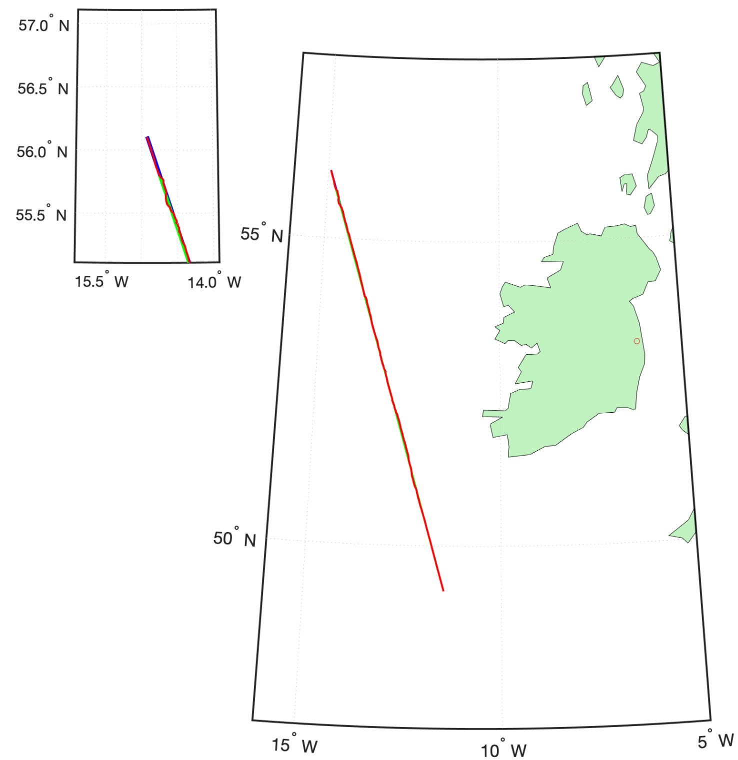
Blue – Ground truth
Red – Standard INS
Green – INS w/Grav Grad Map-Matching

Example – Irish Sea Data





Example using straight line trajectory and the Irish sea data.
The horizontal errors (left) for the augmented navigation solution (red) and inertial navigation alone (green).
Straight line trajectory and the Irish sea data (right) with true trajectory (blue), augmented trajectory (red), and inertial navigation solution only (blue).

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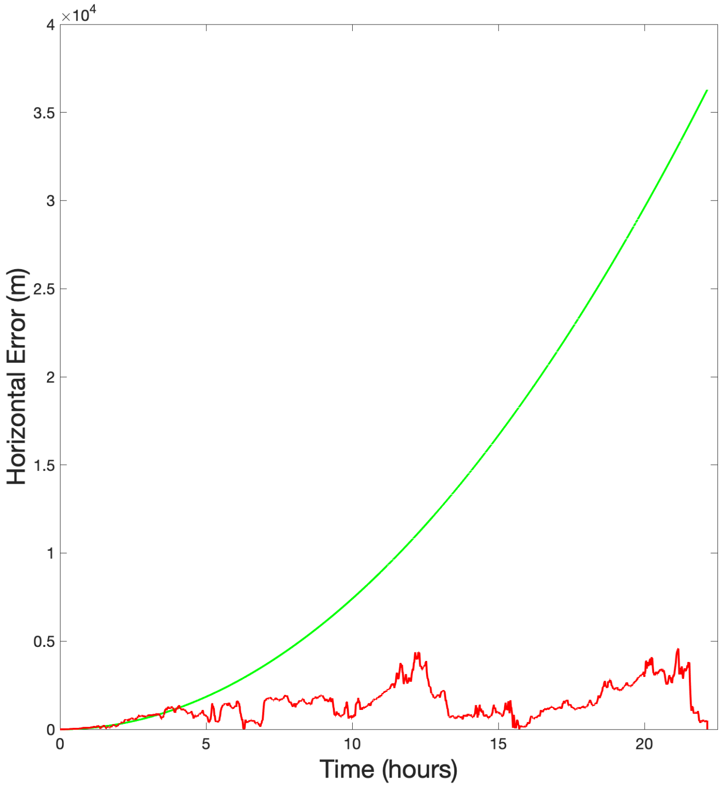
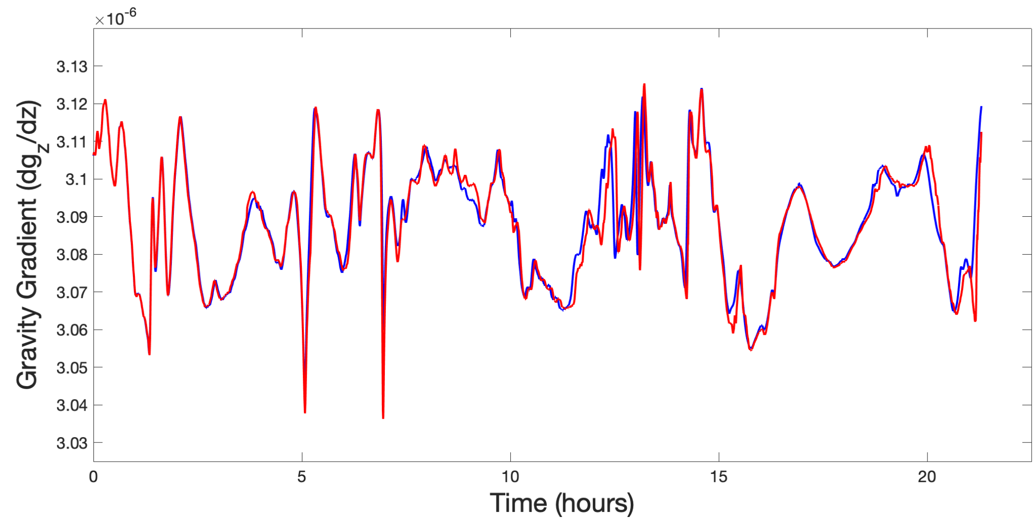
Example – Irish Sea Data

Blue – Ground truth
Red – Standard INS
Green – INS w/Grav Grad Map-Matching





Example using straight line trajectory and the Irish sea data.
The horizontal errors (left) for the augmented navigation solution (red) and inertial navigation alone (green).
The estimated gravity gradients (right) provided by the particle filter (red) and the actual database values (blue)

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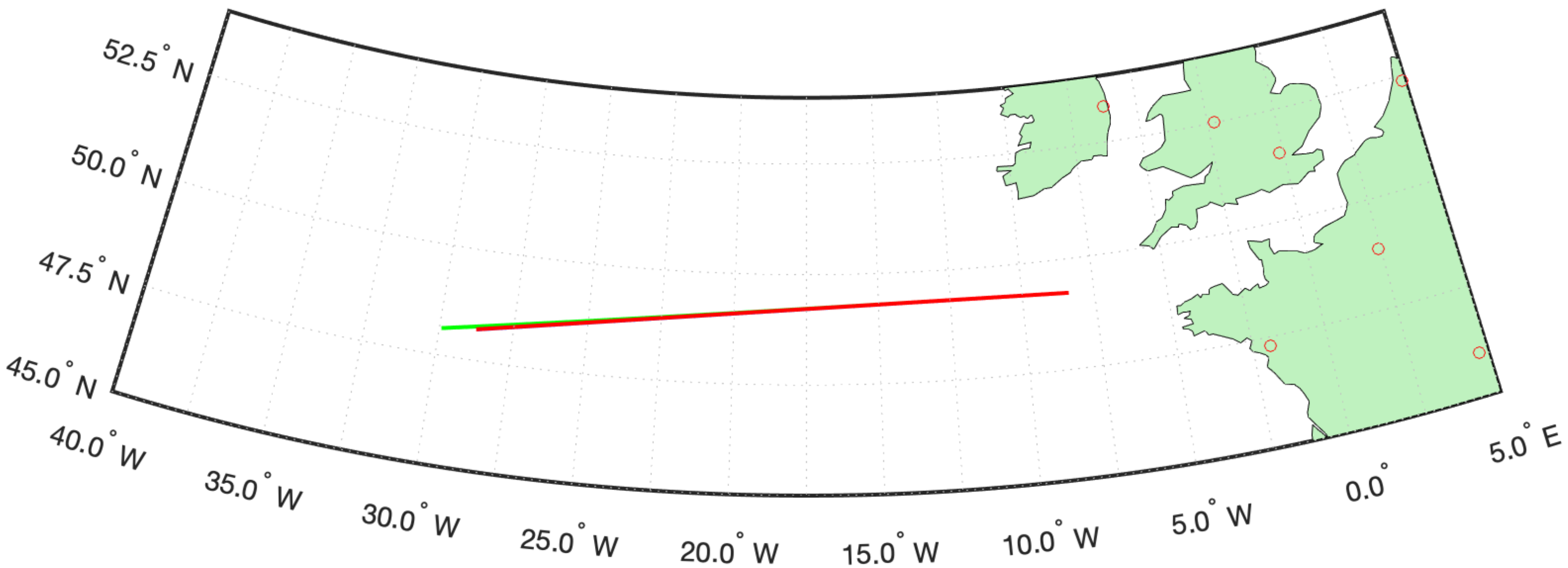
Example – EGM2008 Data

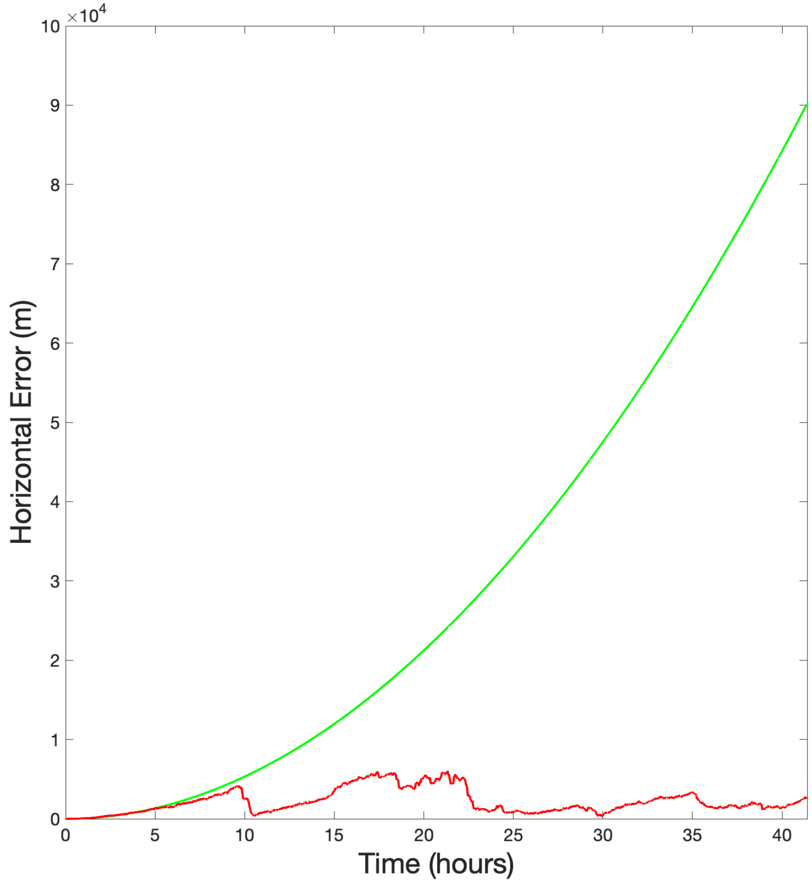
Blue – Ground truth
Red – Standard INS
Green – INS w/Grav Grad Map-Matching



Example straight line trajectory using the EGM2008 database with true trajectory (blue), augmented trajectory (red), and inertial navigation solution only (green).

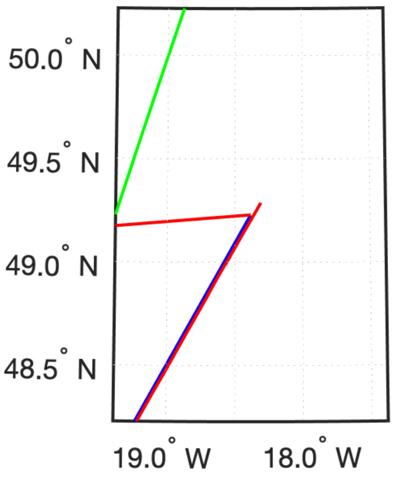
Example – EGM2008 Data

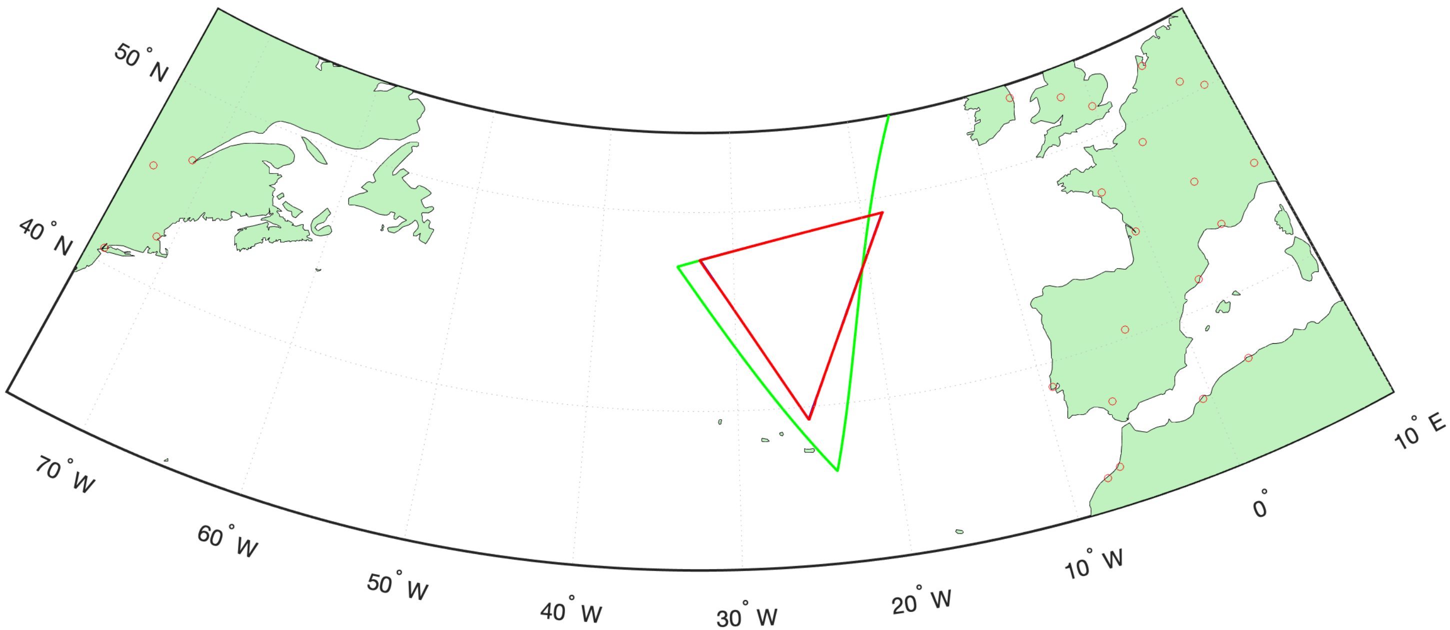
Red – Standard INS
Green – INS w/Grav Grad Map-Matching



The horizontal errors for the augmented navigation solution (red) and inertial navigation alone (green).

Example – Simulated 91 Hour Triangular Trajectory, EGM2008 Data





Blue – Ground truth
Red – Standard INS
Green – INS w/Grav Grad Map-Matching

Summary and Conclusions

- Navigation is complicated by instabilities
  - Long timescales and distances make things worse, but they are not the root cause of the problems
  - Dead reckoning **needs** augmentation – even if you had perfect sensors
- Navigation systems are **not linear**, and they are **not perturbative**
- Augmentation is often susceptible to spoofing and jamming
- Quantum Sensing is the Answer! 🤔
- New processing method for cold atom gradiometers
  - Provides a natural method to integrate with and to augment INS systems
- Gravity gradient map-matching allows navigation that is:
  - Autonomous, Passive, and Impossible to jam or to spoof

Thanks to partners

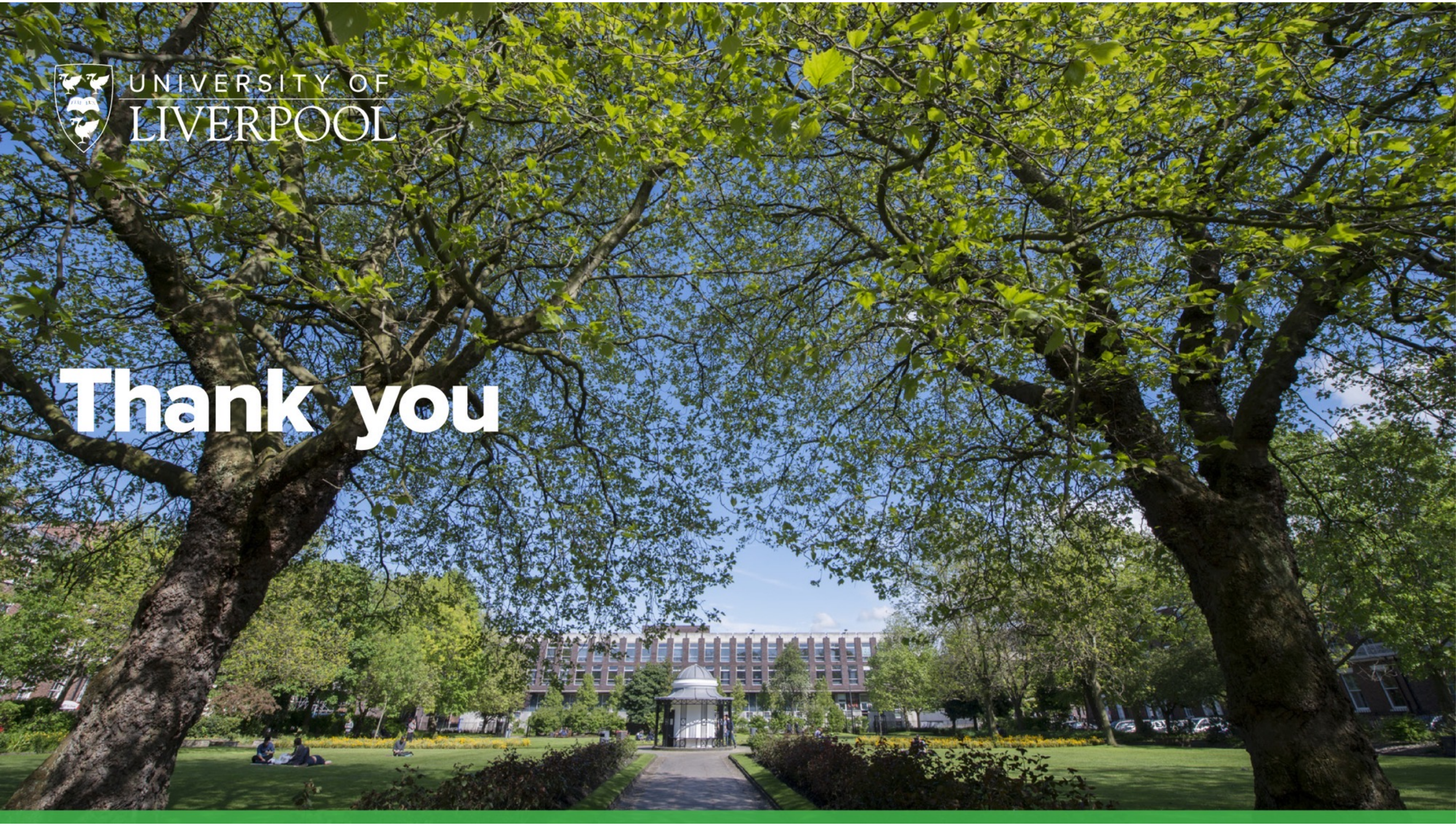
TELEDYNE | Teledyne e2v

esa

Raytheon UK

[dstl]
The Science Inside

ULTRA


UNIVERSITY OF LIVERPOOL
Thank you