NAVAL POSTGRADUATE SCHOOL

Signatures of Braking Surface Targets in Spotlight Synthetic Aperture Radar

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Executive Summary

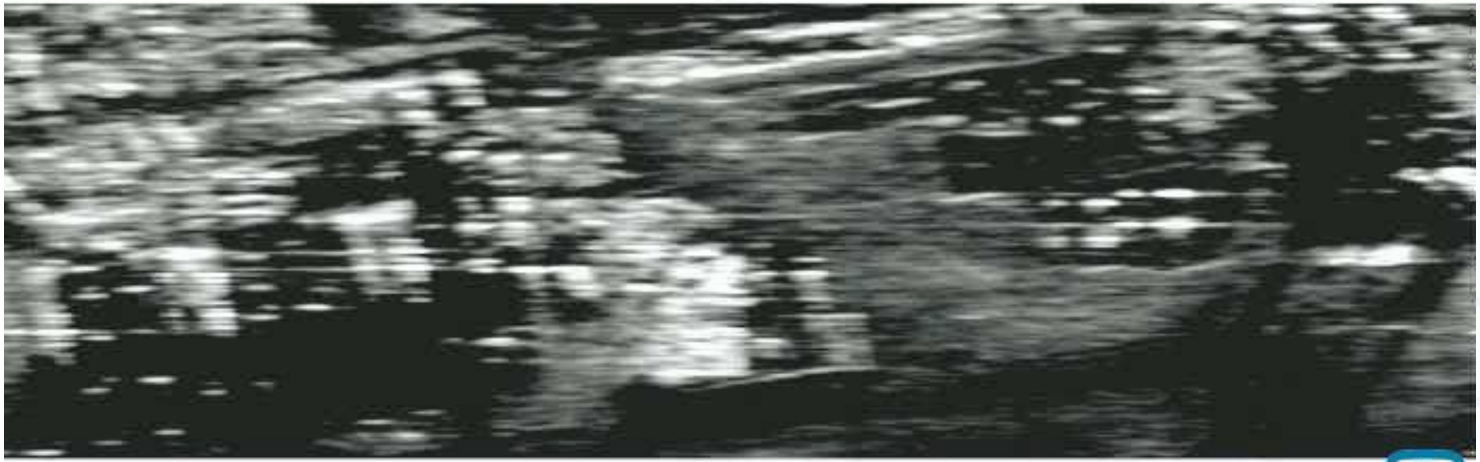
- This analysis examines the signature characteristics of moving targets in synthetic aperture radar (SAR) imagery
- This analysis considers the special case in which the radar sensor is assumed to move with constant speed and heading on a level flight path with broadside imaging geometry
- Resulting defocused signature in the spotlight SAR image is primarily smeared in the radar cross-range direction, although there is smearing in the down-range direction
- Cases of uniform target motion exhibit simply curved target smear shapes that are approximately parabolas
- Other types of motion can give complicated smear shapes
- Analytic equations have been developed to predict these complicated smear shapes

SAR Motion Compensation

SAR Motion Compensation Removes Blurring Due to Phase Errors

Motion Compensated

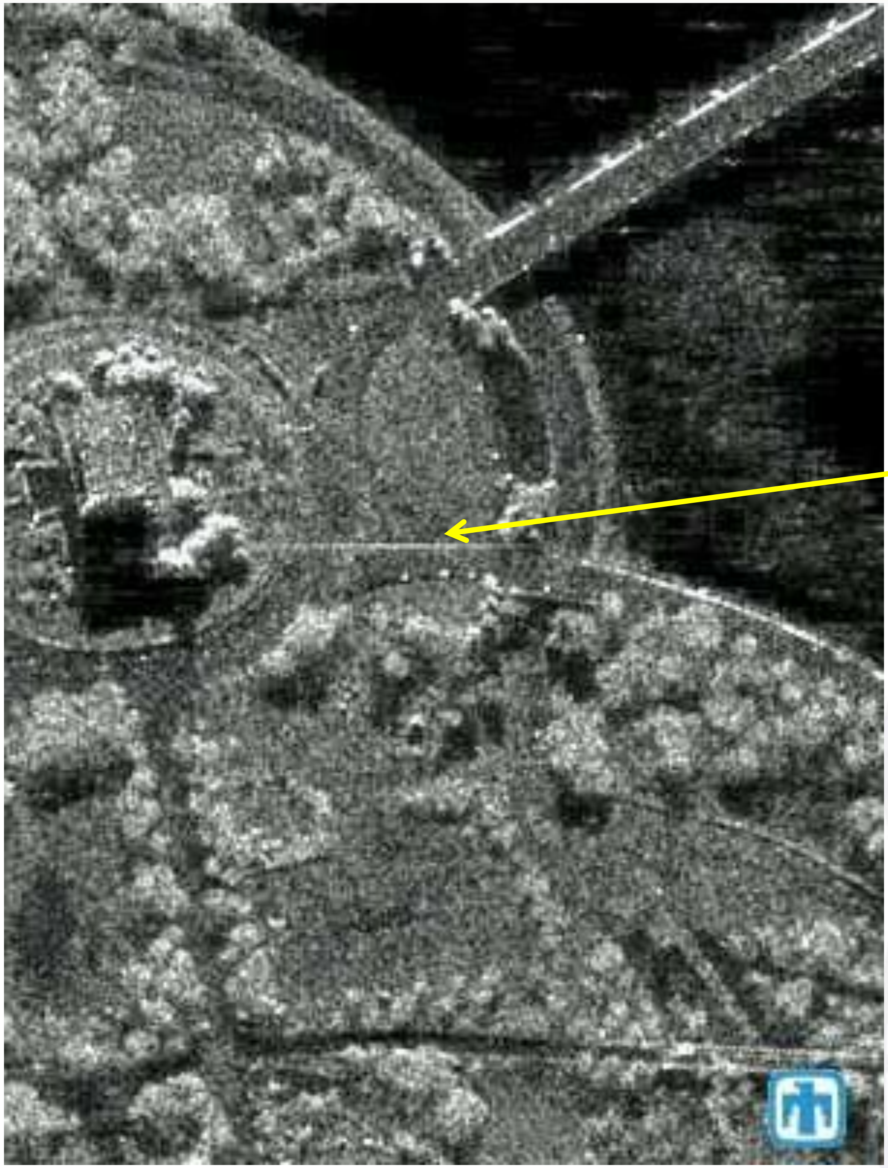
Uncompensated



Ku-band, 3-meter resolution

3 Meter Resolution

Moving Targets in SAR Imagery



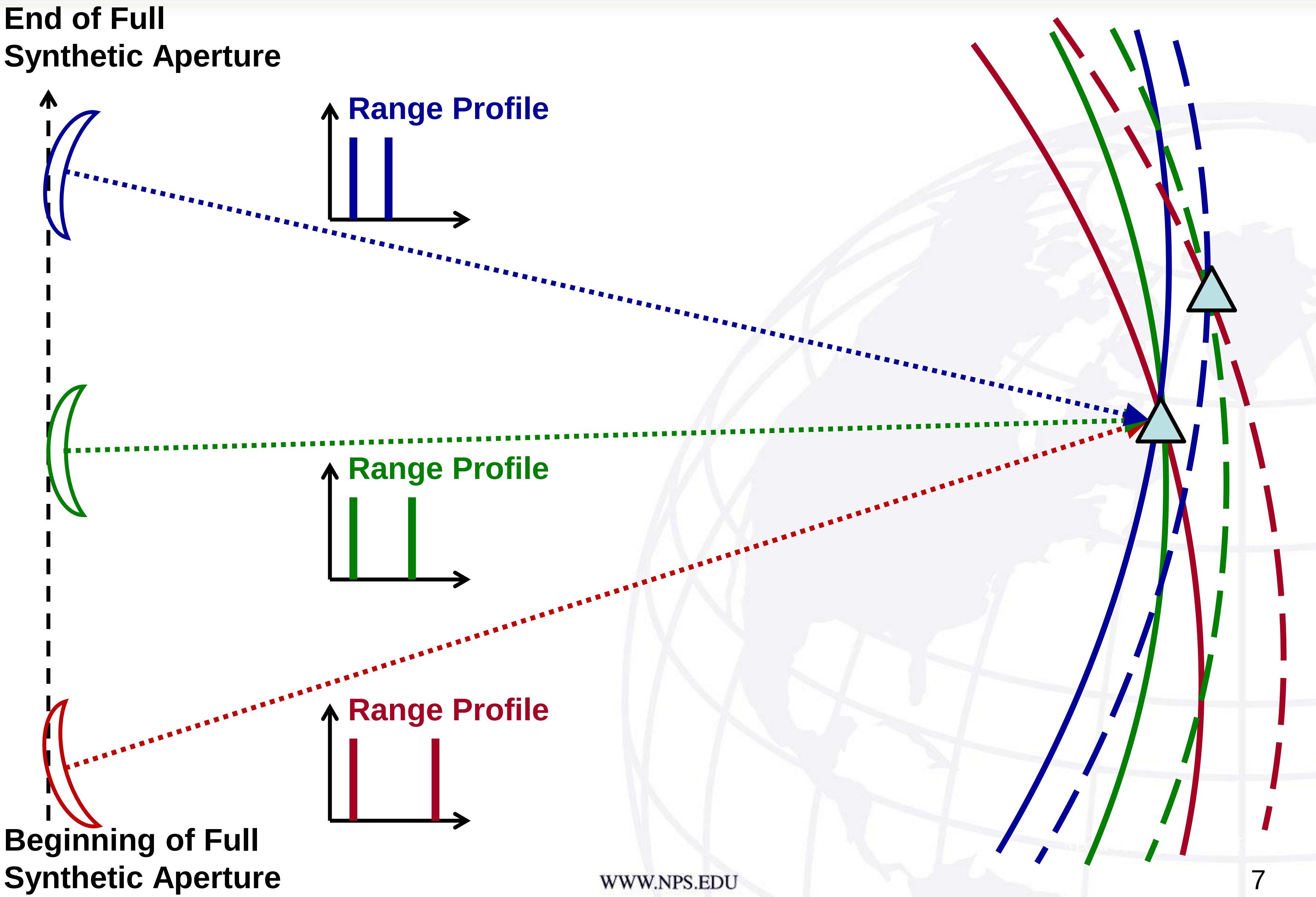
- SAR image formation is effectively a matched filtering process assuming radar scattering off of stationary points within the scene
- Moving targets are not correctly matched to the SAR image formation process and thus display as defocused smears in the imagery
- Current research attempts to understand the shapes of these moving target signatures from a theoretical perspective

Smear Artifacts in SAR

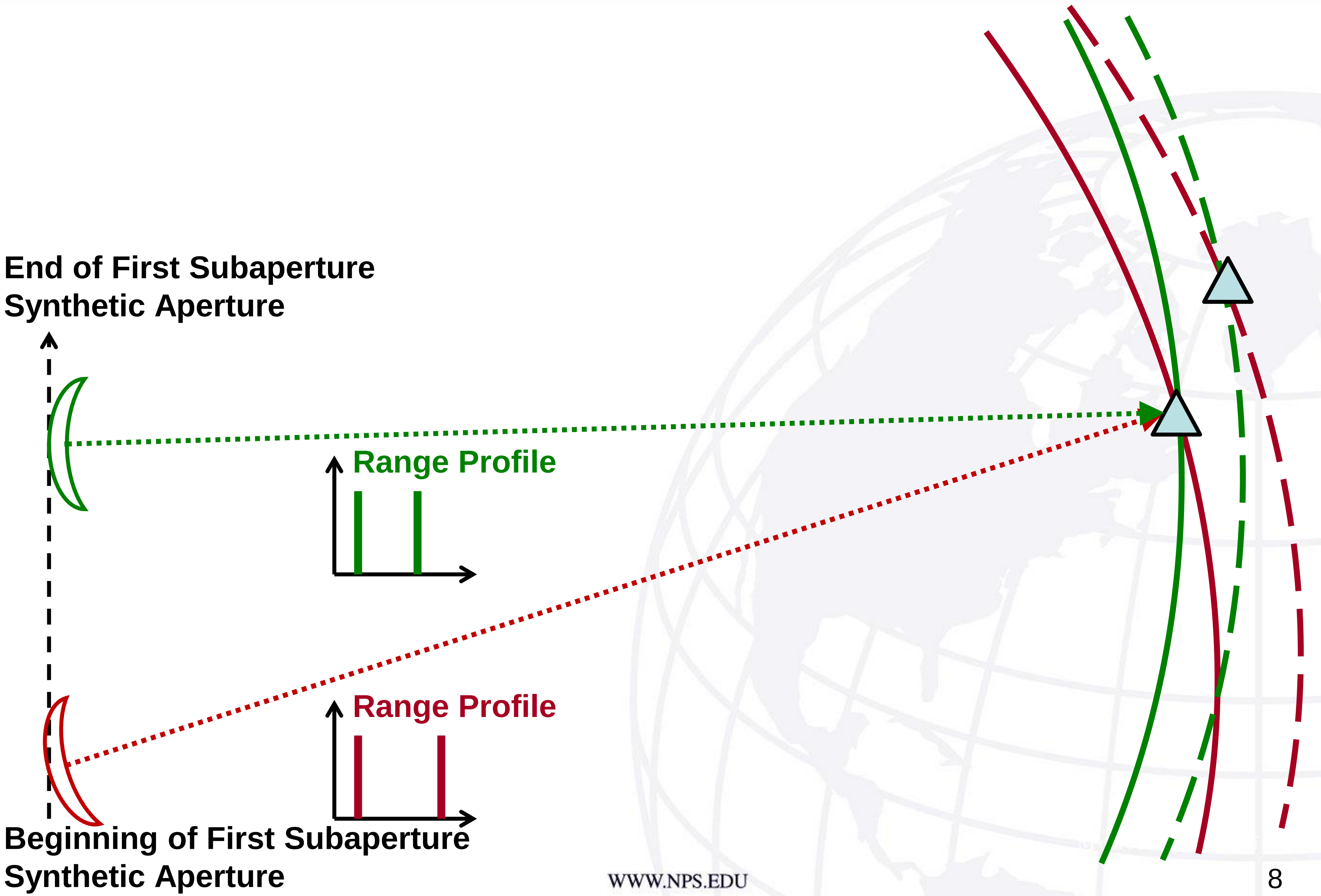
SAR Image of Pentagon Shows Several Smear Artifacts

Analytics of Moving Target Signatures in SAR Imagery

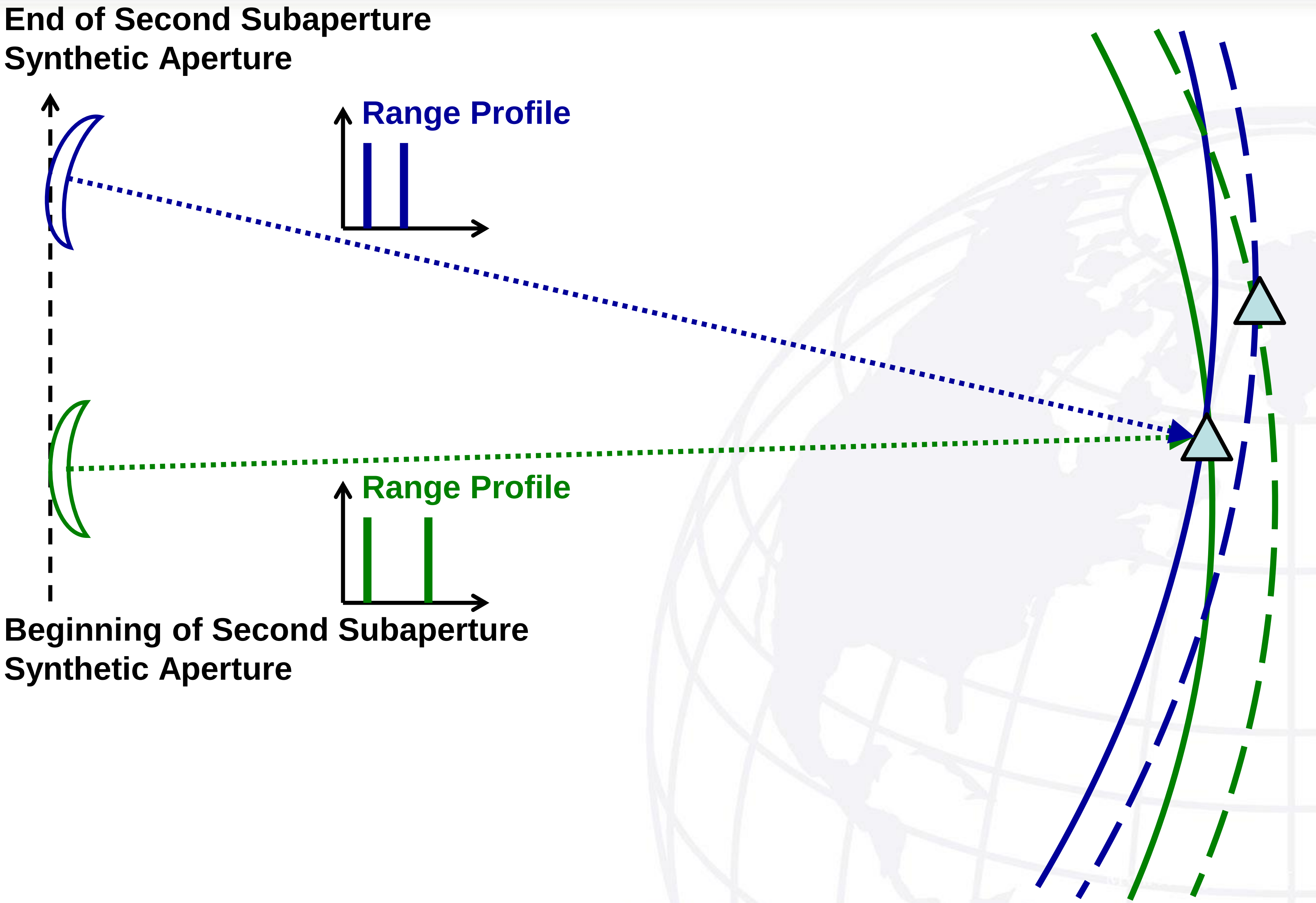
Range Profiles at Different Angles



Subaperture Image Formation



Subaperture Image Formation



Subaperture SAR Images

- Subaperture SAR images are computed via:

$$b_s(x,y) = \int_{\xi_0-\Delta\xi/2}^{\xi_0+\Delta\xi/2} d\xi \int_{\eta_s-\Delta\eta/2}^{\eta_s+\Delta\eta/2} d\eta\, \tilde{G}(\xi,\eta)\exp\big(j2\pi(x\xi+y\eta)\big)$$

- s = subaperture index
- $\xi_0 = 2f_c/c$ = central ground range spatial frequency
  - f_c = central value of the waveform temporal frequency
- η_s = central ground cross-range spatial frequency
- $\Delta\xi = 2\Delta f/c =$ ground range spatial frequency bandwidth
  - $\Delta f =$ waveform temporal frequency bandwidth
- $\Delta\eta$= ground cross–range spatial frequency bandwidth
- $\tilde{G}(\xi,\eta)$ = target response function obtained from the radar measurements

Target Response Function

- Response function $\tilde{G}(\xi,\eta)$ for a single moving point target:

$$b_s(x,y) = \int_{\xi_0-\Delta\xi/2}^{\xi_0+\Delta\xi/2} d\xi \int_{\eta_s-\Delta\eta/2}^{\eta_s+\Delta\eta/2} d\eta\, \tilde{G}(\xi,\eta)\exp(j2\pi\{x\xi + y\eta\})$$

$$\tilde{G}(\xi,\eta) = \sigma_0 \exp\big(-j2\pi\{\alpha_s\big(t(\xi,\eta)\big)\xi + \beta_s\big(t(\xi,\eta)\big)\eta\}\big)$$

- $\sigma_0 =$ complex-valued scattering response of moving target
- $\alpha_s(t) =$ instantaneous down-range position of moving target
- $\beta_s(t) =$ instantaneous cross-range position of moving target
- These equations require knowledge of the slow-time t for every sample value of spatial frequency coordinates $\{\xi,\eta\}$
  - Can be computed using knowledge of radar platform trajectory

Phase Function

- Subaperture image function $b_s(x,y)$ has the form:

$$b_s(x,y) = \sigma_0 \int_{-\Delta\xi/2}^{\Delta\xi/2} d\tilde{\xi} \int_{-\Delta\eta/2}^{\Delta\eta/2} d\tilde{\eta} \exp\left(\Omega\left(x,y,\tilde{\xi},\tilde{\eta},\eta_s\right)\right)$$

- A phase function $\Omega\left(x,y,\tilde{\xi},\tilde{\eta},\eta_s\right)$ has been defined by:

$$\Omega\left(x,y,\tilde{\xi},\tilde{\eta},\eta_s\right)$$

$$\equiv \left\{x-\alpha_s\left(t\left(\tilde{\xi},\tilde{\eta},\eta_s\right)\right)\right\}\left\{\xi_0+\tilde{\xi}\right\}+\left\{y-\beta_s\left(t\left(\tilde{\xi},\tilde{\eta},\eta_s\right)\right)\right\}\left\{\eta_s+\tilde{\eta}\right\}$$

- This function will be expanded in terms of a power series analysis in order to elucidate the primary signature characteristics of moving targets in SAR imagery

Three Small Parameters

- To further understand moving target signatures in spotlight SAR imagery, it is useful to perform a power series expansion of the phase function $\Omega\left(x, y, \tilde{\xi}, \tilde{\eta}, \eta_s\right)$ in terms of three dimensionless parameters that are typically small for narrow-band SAR image formation:

$$\varepsilon_x \equiv \frac{\tilde{\xi}}{\xi_0}, \qquad \varepsilon_y \equiv \frac{\tilde{\eta}}{\xi_0}, \qquad \varepsilon_s \equiv \frac{\eta_s}{\xi_0}$$

- Analysis of the subaperture image function $b_s(x, y)$ requires evaluation of the phase $\Omega\left(x, y, \tilde{\xi}, \tilde{\eta}, \eta_s\right)$ through at least third order in the three-dimensional Taylor series expansion in terms of $\{\varepsilon_x, \varepsilon_y, \varepsilon_s\}$ to assess the signature characteristics of moving scattering centers within the SAR imagery

Method of Stationary Phase

- It is desirable to find numeric expressions for the central or peak location of the smear signatures within each of the subaperture images
- By the method of stationary phase, the integral in $b_s(x,y)$ has a maximum value when the coefficients of the dominant phase terms in $\Omega\left(x,y,\varepsilon_x,\varepsilon_y,\varepsilon_\mathrm{s}\right)$ are set equal to zero
- Due to the power series expansion of $\Omega\left(x,y,\varepsilon_x,\varepsilon_y,\varepsilon_s\right)$, the dominant order terms that remain within the integral for $b_s(x,y)$ are terms that are linear, i.e., the ε_x and ε_y terms
- Thus the application of the method of stationary phase requires that the coefficients of ε_x and ε_y both be set equal to zero independently in order to find constraints on the location of the intensity peak in the smear signature location

Analytic SAR Signature Equations

$$\frac{\Omega\left(x,y,\varepsilon_x,\varepsilon_y,\varepsilon_s\right)}{\xi_0}=\left\{\left[x-\mu_0\left(\tau_s\right)\right]+\varepsilon_s\left[y-\nu_0\left(\tau_s\right)\right]\right\}+\varepsilon_x\left\{\left[x-\mu_0\left(\tau_s\right)\right]+\varepsilon_s\kappa_0\mu_1\left(\tau_s\right)+\varepsilon_s^2\kappa_0\nu_1\left(\tau_s\right)\right\}$$
$$+\varepsilon_y\left\{\left[y-\nu_0\left(\tau_s\right)\right]-\kappa_0\mu_1\left(\tau_s\right)-\varepsilon_s\kappa_0\nu_1\left(\tau_s\right)\right\}$$
$$+\varepsilon_x\varepsilon_y\varepsilon_s\left\{2\kappa_0\nu_1\left(\tau_s\right)+\kappa_0^2\mu_2\left(\tau_s\right)\right\}-\varepsilon_y^2\left\{\kappa_0\nu_1\left(\tau_s\right)+\frac{1}{2}\kappa_0^2\mu_2\left(\tau_s\right)+\varepsilon_s\frac{1}{2}\kappa_0^2\nu_2\left(\tau_s\right)\right\}$$
$$+\varepsilon_x\varepsilon_y^2\left\{\kappa_0\nu_1\left(\tau_s\right)+\frac{1}{2}\kappa_0^2\mu_2\left(\tau_s\right)\right\}-\varepsilon_y^3\left\{\frac{1}{2}\kappa_0^2\nu_2\left(\tau_s\right)+\frac{1}{6}\kappa_0^2\mu_2\left(\tau_s\right)\right\}$$

- Method of stationary phase implies that the central position of a moving target signature smear for a subaperture with mean time τ_s is obtained by setting the dominant linear order terms of ε_x and ε_y terms equal to zero in $\Omega(x,y,\varepsilon_x,\varepsilon_y,\varepsilon_s)$, yielding the following smear trajectory:

Down-Range Component of Target Smear Signature:

$$x\left(\tau_s\right)=\mu_0\left(\tau_s\right)-\varepsilon_s\kappa_0\mu_1\left(\tau_s\right)-\varepsilon_s^2\kappa_0\upsilon_1\left(\tau_s\right)$$

Cross-Range Component of Target Smear Signature:

$$y\left(\tau_s\right)=\upsilon_0\left(\tau_s\right)+\kappa_0\mu_1\left(\tau_s\right)+\varepsilon_s\kappa_0\upsilon_1\left(\tau_s\right)$$

Spotlight SAR Simulations

- Generate simulated smear signatures for moving targets within spotlight SAR imagery
- Radar platform is assumed to be imaging with broadside geometry and a constant and level flight path
- Radar platform speed = 200 m/s
- Radar ground range from ground reference point = 30 km
- Radar platform elevation = 1 km
- Radar center frequency = 1.5 GHz
- Radar bandwidth = 0.15 GHz
- Number of I/Q samples over radar bandwidth = 700
- Total synthetic aperture time = 15 sec
- Number of radar waveforms over synthetic aperture = 1000

Constant Velocity Targets

True Constant Velocity Trajectory

- Initial special case to be examined is that of a constant velocity point target, characterized by:

Down-Range Component of True Target Position:

$$\alpha(t) = \bar{\alpha}_0 + \bar{\alpha}_1 t$$

Cross-Range Component of True Target Position:

$$\beta(t) = \bar{\beta}_0 + \bar{\beta}_1 t$$

- Thus, the coefficient expansion functions have the form:

$$\text{position} = \{\mu_0(\tau_s), \upsilon_0(\tau_s)\} = \{\bar{\alpha}_0, \bar{\beta}_0\}$$

$$\text{velocity} = \{\mu_1(\tau_s), \upsilon_1(\tau_s)\} = \{\bar{\alpha}_1, \bar{\beta}_1\}$$

Constant Velocity Smear Signature

- Use coefficient expansion function:

$$\text{position} = \left\{\mu_0\left(\tau_s\right), \upsilon_0\left(\tau_s\right)\right\} = \left\{\bar{\alpha}_0, \bar{\beta}_0\right\}$$

$$\text{velocity} = \left\{\mu_1\left(\tau_s\right), \upsilon_1\left(\tau_s\right)\right\} = \left\{\bar{\alpha}_1, \bar{\beta}_1\right\}$$

- Combine with general smear signature equations:

$$x\left(\tau_s\right) = \mu_0\left(\tau_s\right) - \varepsilon_s\kappa_0\mu_1\left(\tau_s\right) - \varepsilon_s^2\kappa_0\upsilon_1\left(\tau_s\right)$$

$$y\left(\tau_s\right) = \upsilon_0\left(\tau_s\right) + \kappa_0\mu_1\left(\tau_s\right) + \varepsilon_s\kappa_0\upsilon_1\left(\tau_s\right)$$

- Gives the specific smear signature equations for a constant velocity target:

Down-Range Component of Target Smear Signature:

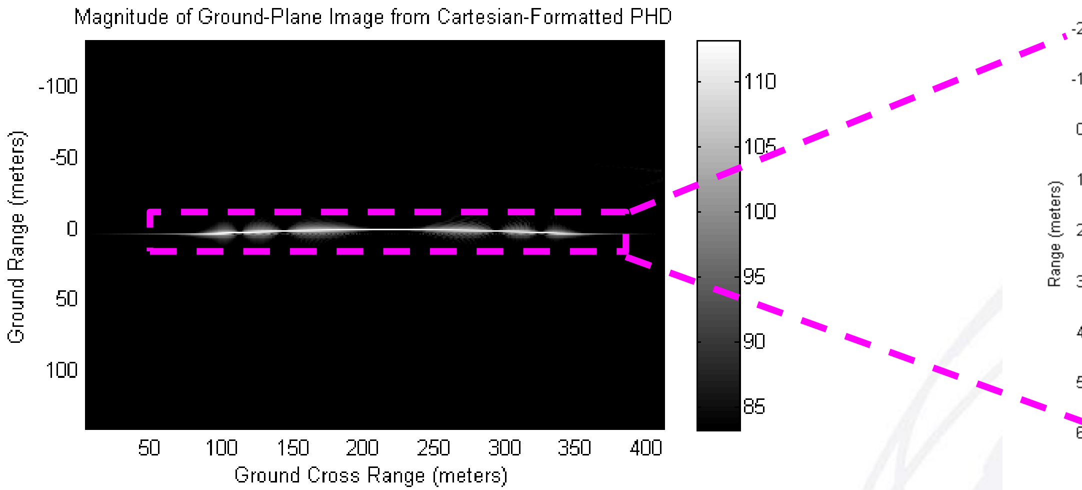
$$x\left(\tau_s\right) = \bar{\alpha}_0 - \frac{\bar{\beta}_1}{\kappa_0}\tau_s^2$$

Cross-Range Component of Target Smear Signature:

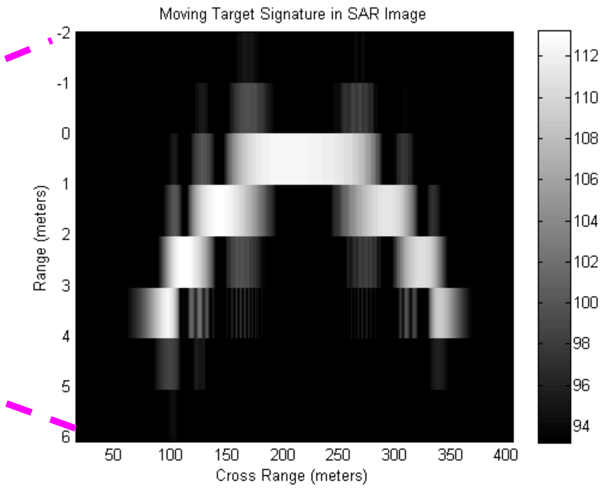
$$y\left(\tau_s\right) = \left\{\bar{\beta}_0 + \kappa_0\bar{\alpha}_1\right\} + 2\bar{\beta}_1\tau_s$$

Constant Velocity Target in SAR
Leftward Pointing Radar Beam

Signature Smear of a Point Target Moving with Constant Velocity



Signature Curvature is More Evident with Unequal Pixel Axes



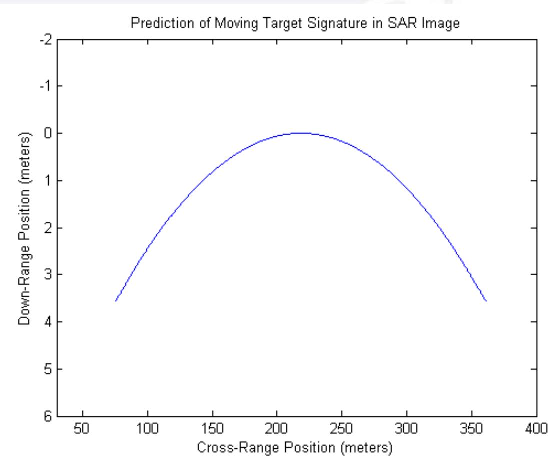
Theoretical calculations correctly predict the location, extent, and shape of the moving target signature smear

$$x(\tau_s) = x_0 - \frac{v_y}{\kappa_0}\tau_s^2$$

$$y(\tau_s) = \{y_0 + \kappa_0 v_x\} + 2v_y\tau_s$$

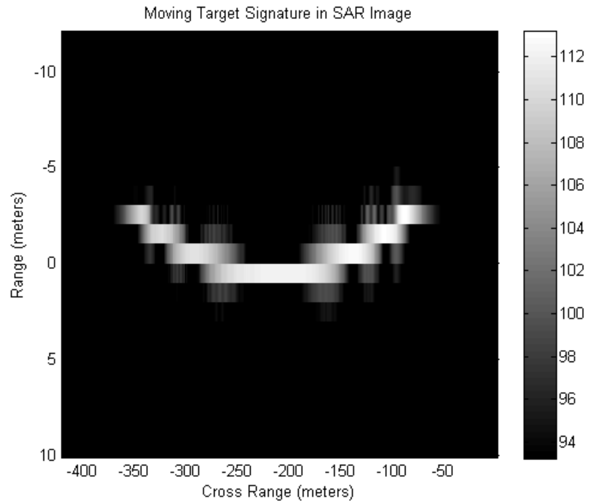
$$\kappa_0 \equiv \mp \frac{X_0}{V_0}$$

minus sign = rightward pointing
positive sign = leftward pointing



Constant Velocity Target in SAR
Rightward Pointing Radar Beam

Radar motion direction has changed sign, so radar mainbeam points to the right



Concavity of the smear flips depends upon the side to which the radar beam points

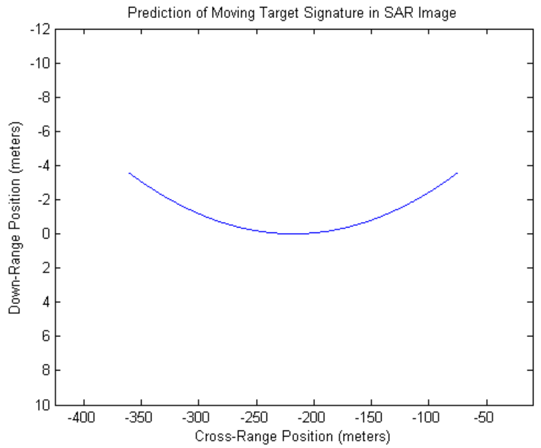
Again, theoretical calculations correctly predict the location, extent, and shape of the moving target signature smear

$$x(\tau_s) = x_0 - \frac{v_y}{\kappa_0}\tau_s^2$$

$$y(\tau_s) = \{y_0 + \kappa_0 v_x\} + 2v_y\tau_s$$

$$\kappa_0 \equiv \mp \frac{X_0}{V_0}$$

minus sign = rightward pointing
positive sign = leftward pointing

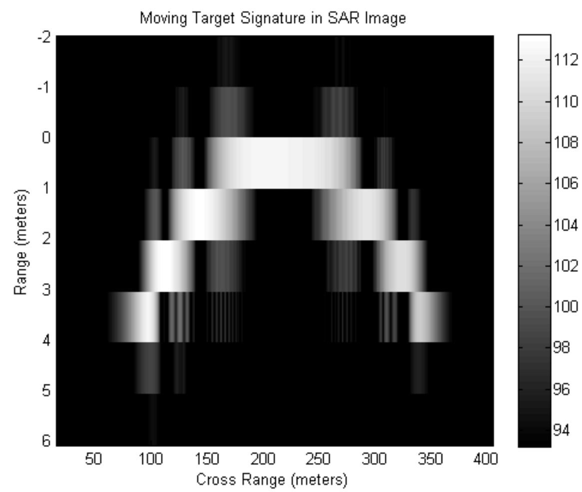


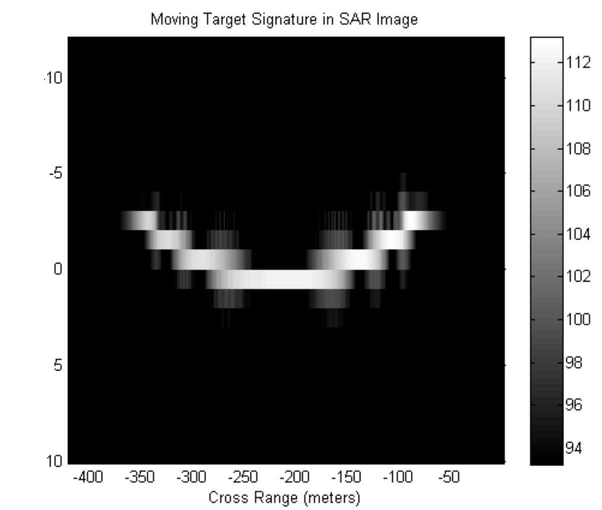
Constant Velocity Target in SAR
Comparison of Radar Beam Direction

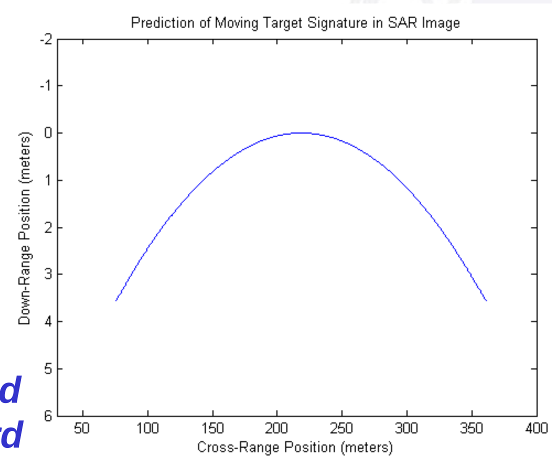
Leftward Pointing Radar Beam

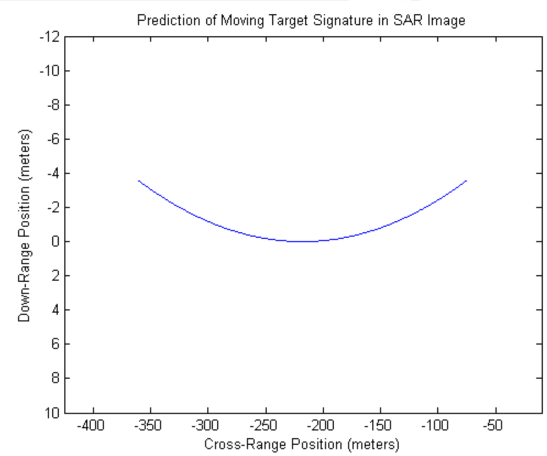
Rightward Pointing Radar Beam

Concavity of the smear flips depending upon which side that the radar beam points









Theoretical calculations:

$$x(\tau_s) = x_0 - \frac{v_y}{\kappa_0}\tau_s^2$$

$$y(\tau_s) = \{y_0 + \kappa_0 v_x\} + 2v_y\tau_s$$

$$\kappa_0 \equiv \mp\frac{X_0}{V_0}$$

minus = rightward
positive = leftward

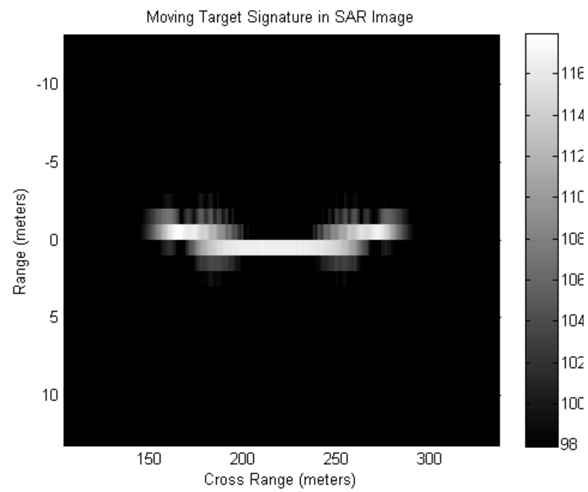
Constant Turning Targets

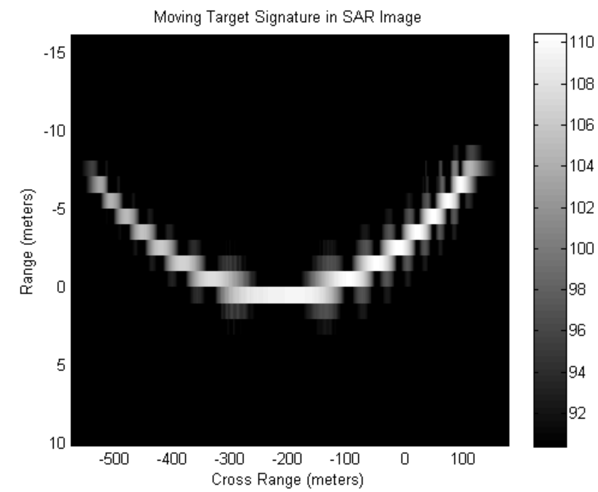
Constant Turning Target in SAR
Comparison of Radar Beam Direction

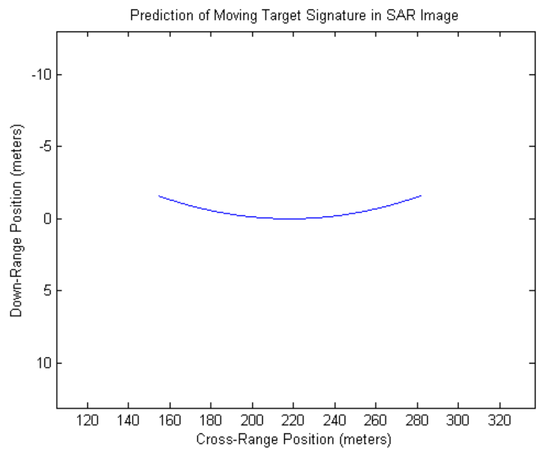
Leftward Pointing Radar Beam | **Rightward Pointing Radar Beam**

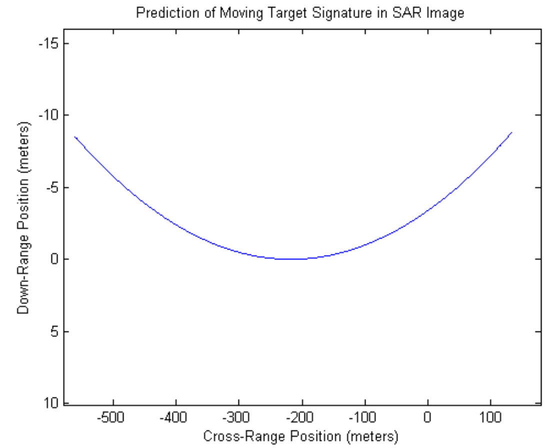
Smear extent is reduced for the leftward pointing case, since the velocity and acceleration terms are partially offsetting

Smear extent is enhanced for the rightward pointing case, since the velocity and acceleration terms are additive





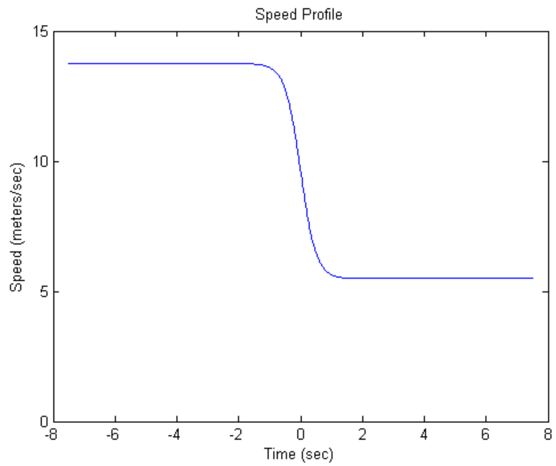




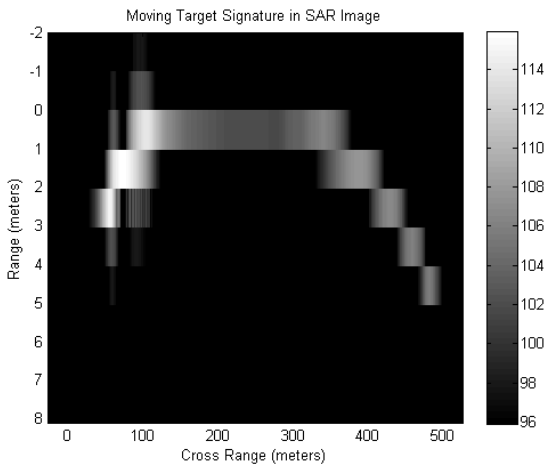
Moving Target Undergoing Braking Maneuver

Target Undergoing Braking Maneuver
Leftward Pointing Radar Beam

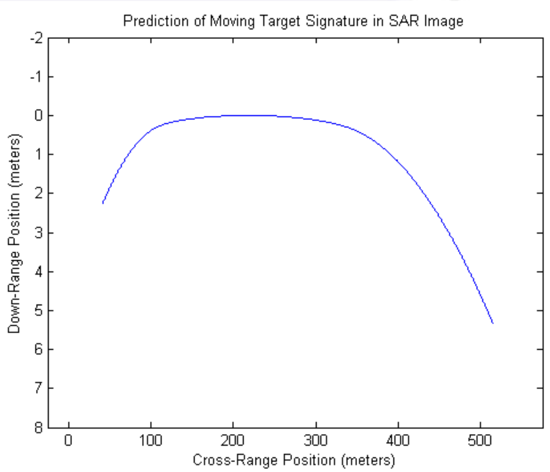
Speed Profile for a Point Target that is Braking While Maintaining Constant Heading



Leftward Pointing Radar Beam



Theoretical Calculations Predict Location, Extent, and Shape



Target Undergoing Braking Maneuver
Rightward Pointing Radar Beam

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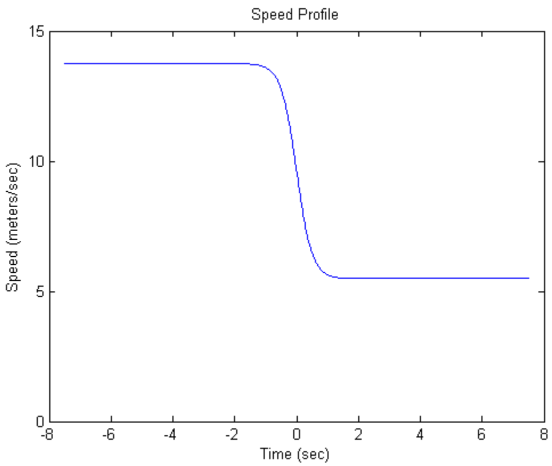
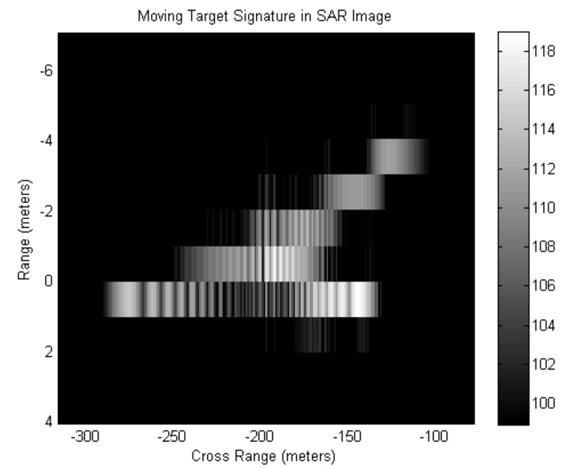
Speed Profile for a Point Target that is Braking While Maintaining Constant Heading



Rightward Pointing Radar Beam



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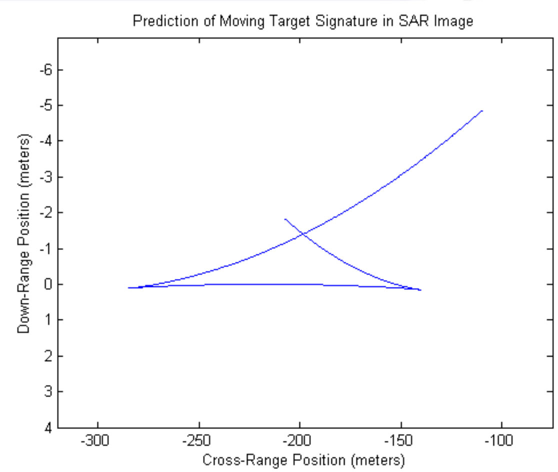

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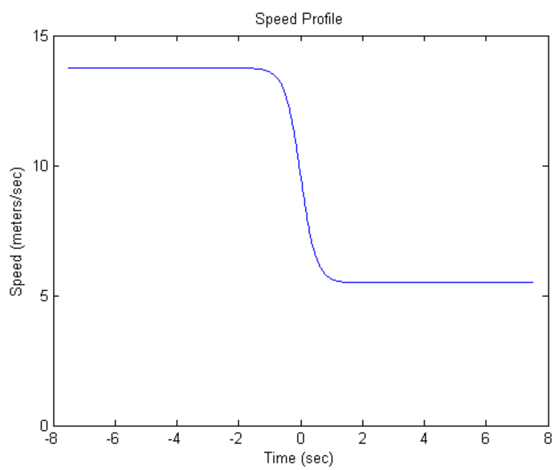
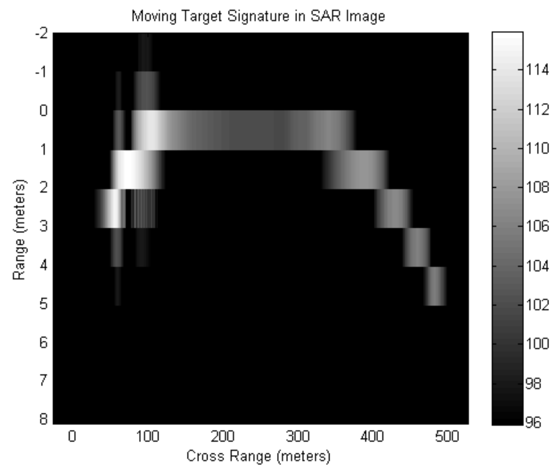
Target Undergoing Braking Maneuver
Comparison of Radar Beam Direction

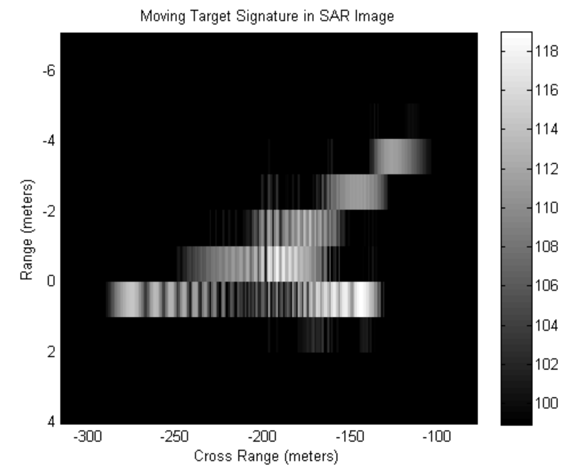
Speed Profile for a Point Target that is Braking While Maintaining Constant Heading



Leftward Pointing Radar Beam



Rightward Pointing Radar Beam



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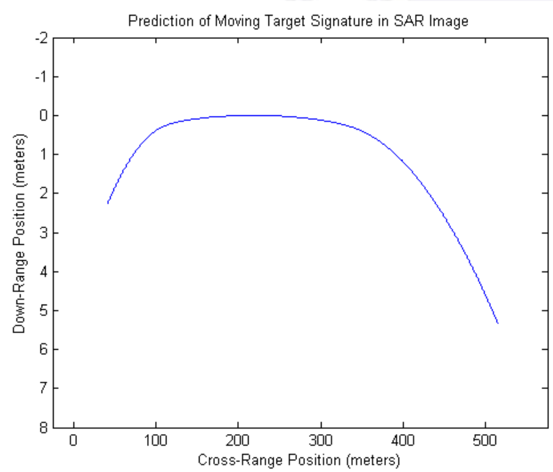

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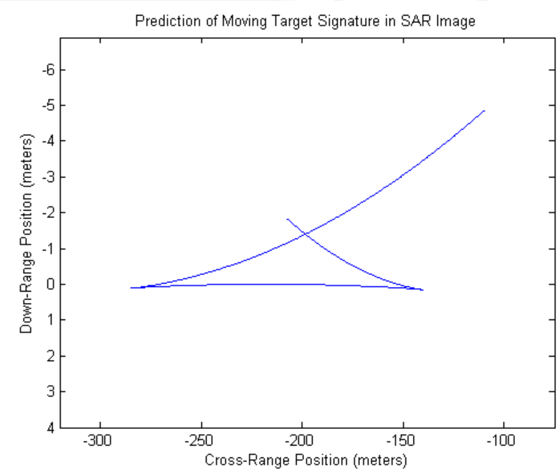
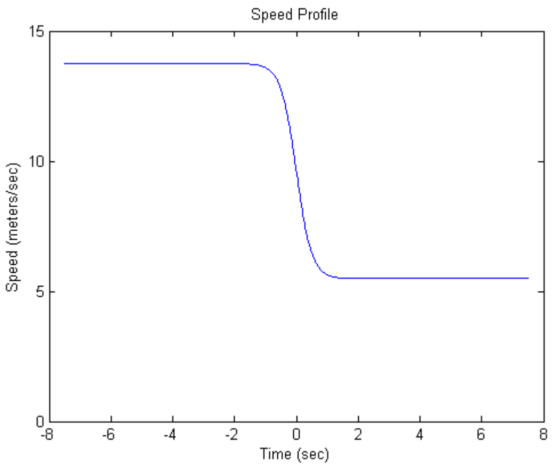
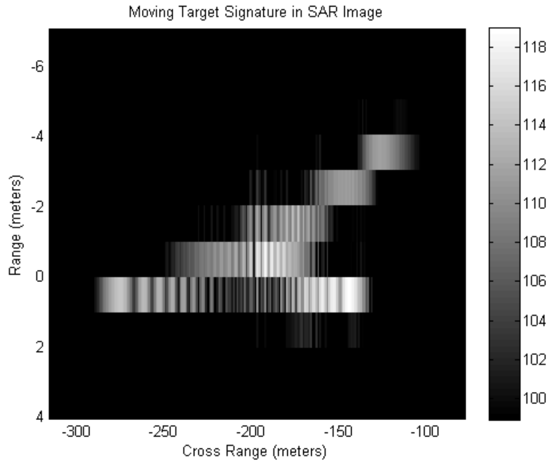


Target Undergoing Braking Maneuver
Components of Smear Signature

Speed Profile for a Point Target that is Braking While Maintaining Constant Heading



Rightward Pointing Radar Beam



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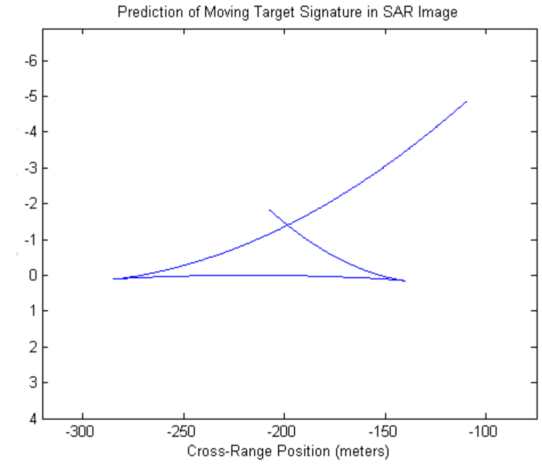
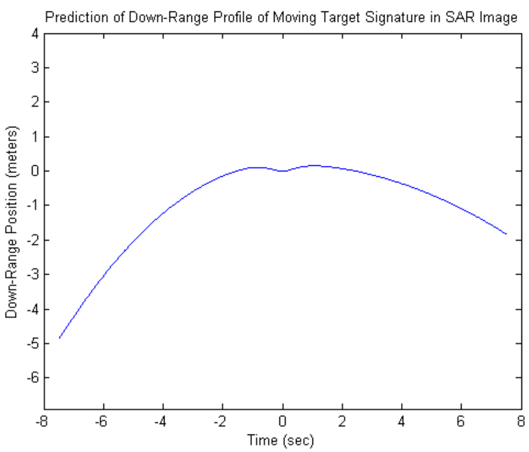
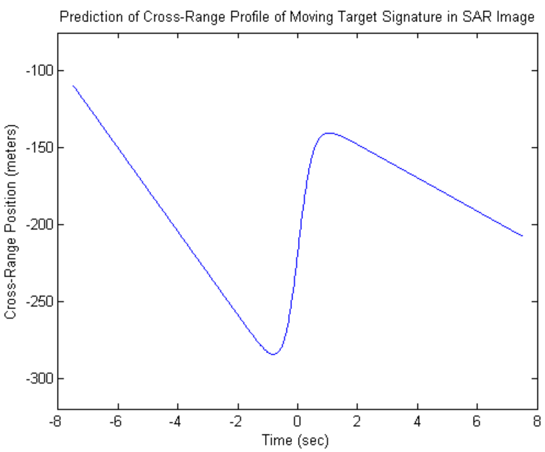


Radar Down-Range Component of Smear Signature



Radar Cross-Range Component of Smear Signature



Summary

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