Truncated unscented particle filter for dealing with non-linear and inequality constraints

Miao Yu and Wen-Hua Chen
Aeronautical and Automotive Engineering Department
Loughborough University, UK, LE11 3TU
Email: {m.yu and w.chen}@lboro.ac.uk

Jonathon Chambers
School of Electronic, Electrical and Systems Engineering
Loughborough University, UK, LE11 3TU
Email: J.A.Chambers@lboro.ac.uk

Introduction

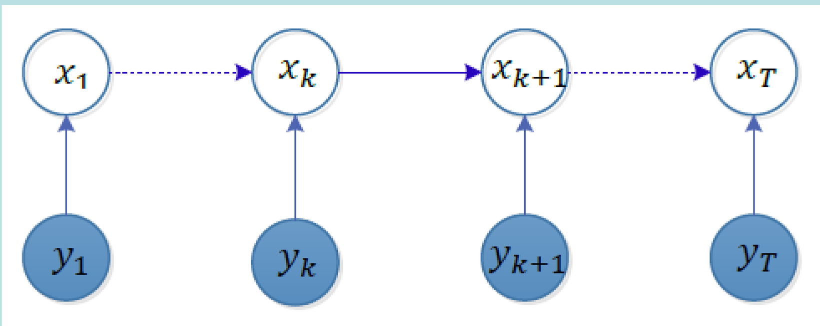
An elegant ***truncated unscented particle filtering scheme*** considering the provided non-linear and inequality constraint information is proposed:

- A ***particle filtering*** method is applied to cope with non-linear models and non-Gaussian state distribution.
- A ***truncated unscented Kalman filter*** is applied as the importance function for sampling new particles.

The advantages of the proposed truncated unscented particle filter algorithm over the state-of-the-art ones are presented by multiple Monte-Carlo simulations.

General constrained tracking problem

Aim: Obtaining the minimum mean square error (MMSE) estimator: $E(x_k|y_{1:k})$



- State model: $x_k = f(x_{k-1}, v_k) \sim p(x_k|x_{k-1})$
- Measurement model: $y_k = h(x_h, e_k) \sim p(y_k|x_k)$

x_k: state variable
y_k: observation from different types of sensors

For the real state estimation problem, some other information is applied to refine the distribution of the state vector x_k

$$p_C(x_k|z_k) \propto \begin{cases} p(x_k|z_k) & \text{if } x_k \in C_k \\ 0 & \text{otherwise} \end{cases}$$

C_k is the feasible area defined as:

$$C_k = \{x_k | x_k \in R^{n_x}, a_k \le C_k(x_k) \le b_k\}$$

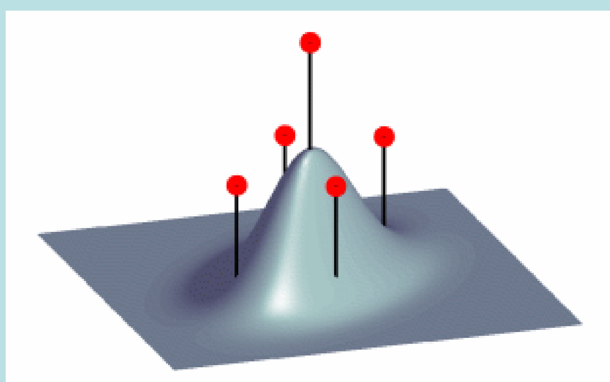
The truncated unscented Kalman filter

The truncated unscented Kalman filter is an extension of the traditional ***unscented Kalman filter*** by considering the ***constraint information***

Initially we have conditional pdf $p_C(x_{k-1}|z_{k-1})$ with mean $\hat{x}_{k-1|k-1}$ and covariance matrix $P_{k-1|k-1}$:

σ-points $\{\chi_{i,k-1|k-1}\}$ and corresponding weights $\{w_{i,k-1|k-1}\}$ are calculated:

$$\chi_{0,k-1|k-1} = \hat{x}_{k-1|k-1} \qquad w_{0,k-1|k-1} = \frac{\kappa}{n_\chi+\kappa}$$
$$\chi_{i,k-1|k-1} = \hat{x}_{k-1|k-1} + \left(\sqrt{(n_\chi+\kappa)P_{k-1|k-1}}\right)_i$$
$$w_{i,k-1|k-1} = \frac{1}{2(n_\chi+\kappa)}$$
$$\chi_{n_\chi+i,k-1|k-1} = \hat{x}_{k-1|k-1} - \left(\sqrt{(n_\chi+\kappa)P_{k-1|k-1}}\right)_i$$
$$w_{n_\chi+i,k-1|k-1} = w_{i,k-1|k-1}$$



According to the σ-points and corresponding weights, the truncated unscented Kalman filter is described as:

- ***Prediction***:

$$\hat{x}_{k|k-1} \approx \sum_i w_{i,k-1|k-1}\, \chi_{i,k|k-1}$$
$$P_{k|k-1} \approx \sum_i w_{i,k-1|k-1} (\chi_{i,k-1|k-1} - \hat{x}_{k|k-1})(\chi_{i,k-1|k-1} - \hat{x}_{k|k-1})^T + Q_{k-1}$$

- ***Correction***:

$$\hat{x}_{k|k} \approx \hat{x}_{k|k-1} + K_{k|k}(z_k - \hat{z}_{k|k-1})$$
$$P_{k|k} = P_{k|k-1} - K_{k|k} P_{z,k|k-1} K_{k|k}^T$$

- ***Importance sampling based probability truncation***

Truncated distribution after the constraints being considered could be approximated as a Gaussian $N_C(x_k|\hat{x}_{k|k}, P_{k|k})$:

$$\hat{x}^c_{k|k} = \frac{1}{N}\sum_i w_k^{c,i}\, x_k^{c,i}$$
$$P^c_{k|k} = \frac{1}{N}\sum_i w_k^{c,i} (x_k^{c,i} - \hat{x}^c_{k|k})(x_k^{c,i} - \hat{x}^c_{k|k})^T$$

- $x_k^{c,i} \in C_k$ and $x_k^{c,i}$ is sampled from a function $q(x)$
- $w_k^{c,i} \propto N(x_k^{c,i}|\hat{x}_{k|k}, P_{k|k})/q(x_k^{c,i})$

The truncated unscented particle filter

Considering that:

- The posterior distribution may not be accurately represented as a single Gaussian due to constraints
- The nonlinear of the state/measurement models

By taking ***the truncated unscented Kalman filter*** as the ***importance function*** for new particles generation, a ***truncated unscented particle filtering scheme*** is proposed :

Initially, a set of particles and weights $\{x^i_{k-1}, w^i_{k-1}\}_{i=1,\ldots,N}$ is applied to approximate $p_C(x_{k-1}|z_{k-1})$

- ***Sampling from the importance function:***

Truncated unscented Kalman filter is used to estimate an importance function $N_C(x_k|\hat{x}_{i,k|k}, P_{i,k|k})$ for every particle i

- ***Sampling and rejection***:

If the obtained sample x^i_k is within the constraint region, the sample is accepted; otherwise, it is rejected.

- ***Weight calculation***:

The weight corresponding to the accepted particle x^i_t is calculated as:

$$w^i_k \propto w^i_{k-1} \frac{p(z_k|x^i_k)p(x^i_k|x^i_{k-1})}{C_i N_C(x^i_k|\hat{x}_{i,k|k}, P_{i,k|k})}$$

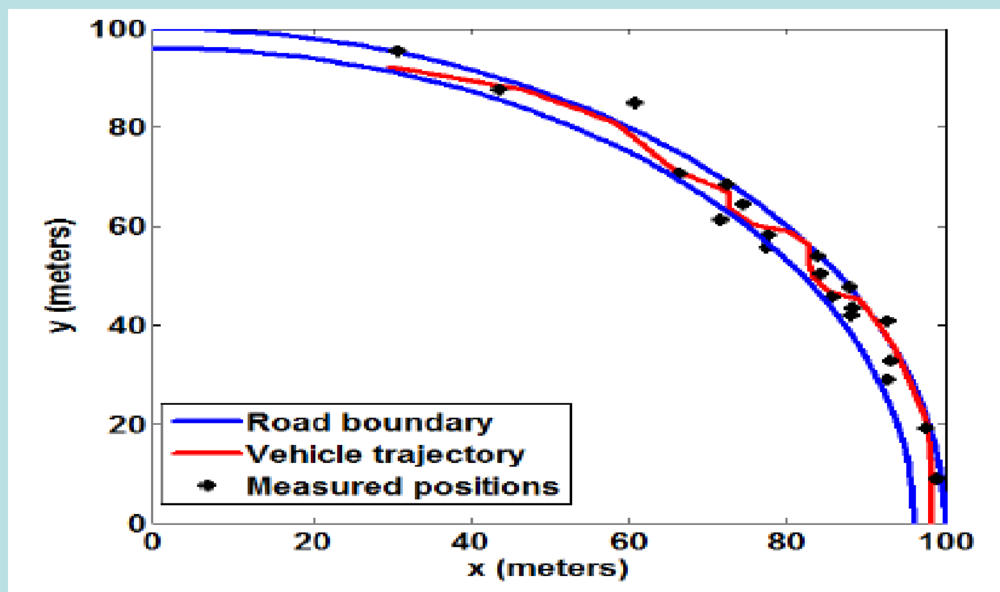
Finally, the weights are summed to one and the state x_t can be estimated as:

$$\hat{x}_k \approx \sum_i w^i_k x^i_k.$$

Simulations

A vehicle is simulated to move with the road constraint:

- The boundaries of the road are defined by two arcs centered at the origin of a Cartesian coordinate system with radius of r1 = 96m and r2 = 100m
- The vehicle dynamics is described by a ***constant velocity model***
- ***Range*** and ***bearing angle*** are measured



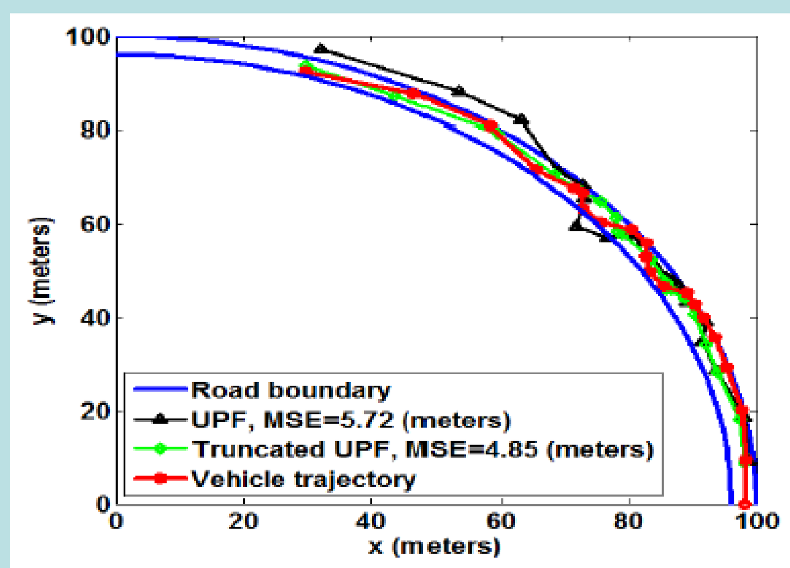
The simulated trajectory of a vehicle moving on a bend road section and the measured positions

Different methods are compared with respect to the mean square errors (MSEs):

The comparison between different methods (100 times Monte-Carlo simulations)

	Accept-rejection	Projection method	Proposed method
Mean of MSEs (meters)	10.89	6.92	5.40
Standard derivation of MSEs (meters)	7.24	2.33	0.91

The comparison results of the T-UPF and standard UPF



Future works

- Considering the `soft' constraints which concern of probability in different regions
- A more realistic scenario will be considered for the miss detection and false alarms, and the algorithm will be developed under a random finite set framework

This work was supported by the Engineering and Physical Sciences Research Council (EPSRC) and Dstl